SQUID Based Superconducting Traveling-Wave Parametric Amplifier

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September 11, 2015 (STH34, HP100). Over the past decade low noise Josephson parametric amplifiers have proven essential in superconducting quantum information science experiments [1]. In parametric amplifiers, high gain is achieved when the signal to be amplified interacts with a nonlinear medium for as long as possible. State-of-the-art Josephson parametric amplifiers utilize resonant circuits to increase the interaction time of the signal with the nonlinear medium. As a consequence, the instantaneous bandwidth and maximum input power allowed are significantly reduced. Such limitations have renewed interest in superconducting traveling-wave parametric amplifiers (TWPAs). Superconducting TWPAs were initially proposed in the early 1980s [2], and have promised high gain, wide bandwidth, and low noise. A practical superconducting TWPA has not been realized to date, mainly because current designs suffer from insufficient gain and excessive noise, largely due to poor phase matching, losses, and weak nonlinearities. Recently there has been renewed interest in superconducting TWPAs mainly from the quantum information sciences community, where a low noise wide bandwidth amplifier capable of multiplexed readout of superconducting qubits is in high demand. The main challenge in TWPA designs is that optimum gain is achieved when phase-matching conditions are met. Superconducting TWPAs have been investigated by many groups, thus far taking one of two approaches: either utilizing a transmission line composed of a series array of Josephson junctions [3] or a transmission line utilizing the nonlinear kinetic inductance of a narrow superconducting wire [4]. These investigations have revealed the need for engineered dispersion to be introduced into the transmission line to facilitate phase matching. Designs which utilize periodic loading and the addition of resonant elements to facilitate phase matching have shown promise, however, at the expense of increased complexity, higher tolerances, and longer propagation lengths (2 cm -1 m).



Fig. 1. Calculated gain of the proposed TWPA. Red and blue lines represent flux tunings (pump powers) of $\Phi/\Phi_0 = 0.45$ (-76 dBm) and $\Phi/\Phi_0 = 0.5$ (-73 dBm) respectively. (a) The signal gain in decibels as a function of the signal frequency. (b) The phase mismatch as a function of the signal frequency. (c) Rendering of the superconducting TWPA composed of a chain of coupled asymmetric SQUIDs as the central conductor of a coplanar transmission line. Arrows represent the electric field being amplified along the length of the transmission line.

We propose an alternative approach and utilize the nonlinear properties of a onedimensional chain of coupled asymmetric superconducting quantum interference devices (SOUIDs) as a transmission line in a TWPA to achieve phase matching and show that exponential gain can be realized over a wide bandwidth, see Figure 1 [5]. The proposed TWPA utilizes the tunable nonlinearity of a one-dimensional chain of asymmetric SQUIDs with nearest-neighbor coupling through mutually shared Josephson junctions as a transmission line to overcome phase-matching limitations. A magnetic flux Φ threads each SQUID to allow for tunability of the linear and nonlinear properties of the transmission line. A weak signal to be amplified and a strong pump tone will be incident on the input port to the transmission line. Because of the nonlinearity of the transmission line, the weak signal will undergo parametric amplification through a degenerate four-wave mixing (FWM) process [6]. The amplification process is the most efficient when the total phase mismatch is close to zero. However, due to the nonlinearity of the transmission line, a strong pump modifies phase matching through self- and cross-phase-modulation, resulting in a phase mismatch. The linear dispersion of the transmission line along with spectral separation of the signal and pump angular frequencies can be used to compensate for the nonlinear phase mismatch. The unique feature of the proposed TWPA is that the linear and nonlinear dispersion can be tuned with Φ , and the nonlinearity can even change sign. By adjusting Φ for a given pump power, phase matching can be achieved.

The design of the proposed TWPA is shown in Fig. 1(c). Each cell of the transmission line is an asymmetric SQUID with a single "small" Josephson junction (blue cross) in one arm and two "large" Josephson junctions (red cross) in the other arm. Fig. 1(a) shows numerical simulations of the signal gain as a function of signal frequency for the proposed TWPA with a transmission line length of 600a (unit-cell length a). For a magnetic flux tuning of $\Phi/\Phi_0 =$ 0.45 and pump power -76 dBm, Figure 1(a) (red line), there are two regions with signal frequency $\omega_s/2\pi = 3.6$ and 9.4 GHz where perfect phase matching $\kappa = 0$ (κ is the sum of the linear and nonlinear contributions to the total phase mismatch) can be achieved, for comparison the phase mismatch dependence on signal frequency is shown in Figure 1(b) (red line). When the exponential gain factor g is real and $\kappa \approx 0$, the gain depends exponentially $\propto \exp(gz)$ on the TWPA length z. When the phase mismatch is the largest at $\omega_s = 6.5$ GHz, g is small and the gain depends quadratically on the length of the TWPA. Due to regions of exponential gain and quadratic gain depending on signal frequency the 3 dB bandwidth of the TWPA ~1.5 GHz is limited to two regions where $\kappa \approx 0$ is centered at $\omega_s/2\pi = 3.9$ and 9.1 GHz. For a magnetic flux tuning of $\Phi/\Phi_0 = 0.5$ and pump power -70 dBm will cause κ to be large Fig. 1(b) (blue line) and only a quadratic gain dependence is possible. Since the signal gain increases quadratically for all frequencies, a relatively flat gain characteristic of the amplifier can be achieved with a signal gain of 23 dB over a 3 dB bandwidth greater than 5.4 GHz.

In summary, a TWPA design based on a chain of coupled asymmetric SQUIDs is presented. The proposed design allows for great flexibility where a magnetic flux can be used to tune the nonlinearity of the transmission line to achieve phase-matching conditions in a four-wave mixing process. Numerical simulations show that the proposed amplifier can achieve signal gains of 23 dB with a 3-dB bandwidth of greater than 5.4 GHz at a center frequency of 6.5 GHz. Under different tuning conditions, gains of greater than 20 dB can be achieved with a relatively short transmission-line length. The proposed amplifier is ideally suited for the multiplexed readout of quantum bits or kinetic-inductance-based astronomical detectors.

References

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