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Modelling and Comparison of In-Field Critical Current Density Anisotropy in High Temperature Superconducting (HTS) Coated Conductors

D Hu[1], M D Ainslie[1], M J Raine[2] D P Hampshire[2] and J Zou[1]

[1]Bulk Superconductivity Group, Department of Engineering, University of Cambridge

[2]Superconductivity Group, Centre for Material Physics, Department of Physics, University of Durham

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Experimental Data Analysis

Experimental Results of Tape Properties from Samples SP1 and SP2

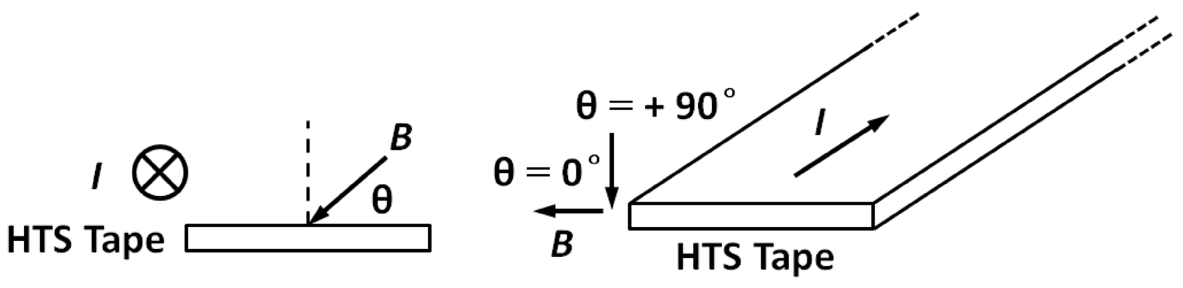

Fig. 1. Definition of the angular, in-field dependence of the critical current density $J_c(B, \Theta)$

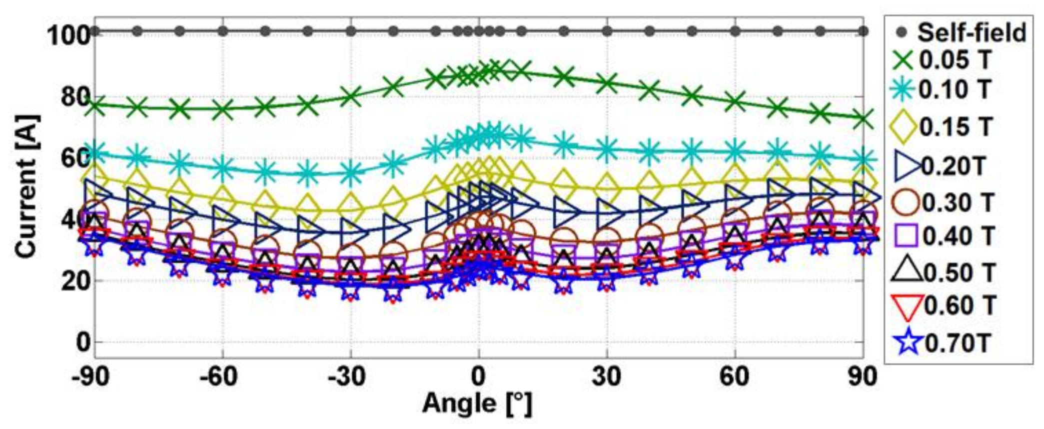

Fig. 2. Comparison of the experimental (symbols) and numerical data fitting (solid lines) for the angular, in-field dependence of the critical current density $J_c(B, \Theta)$ for sample SP1.

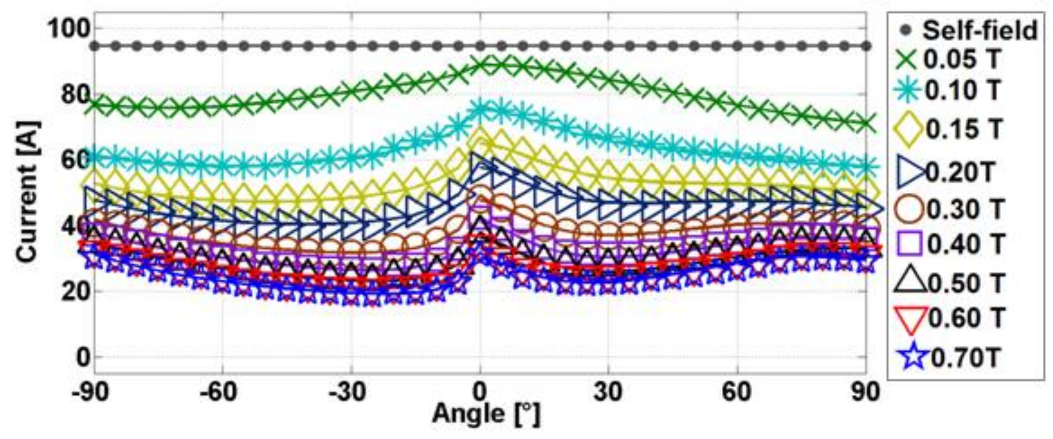

Fig. 3. Comparison of the experimental (symbols) and numerical data fitting (solid lines) for the angular, in-field dependence of the critical current density $J_c(B, \Theta)$ for sample SP2.

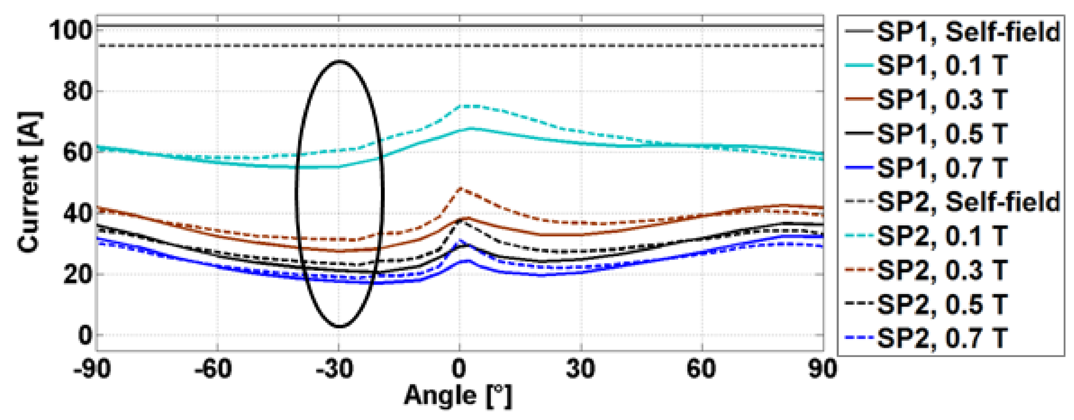

Fig. 4. Comparison of the experimental data for SP1 (solid lines) and SP2 (dashed line) for self-field, and applied fields of 0.1 T, 0.3 T, 0.5 T and 0.7 T.

Data Fitting and Two-Variable Direct Interpolation

Considering the similar trends in these two samples (Fig. 4), an engineering formula can be developed for data fitting to input these data into the numerical model .

$$I_c(B,\theta) = I_{c0} / (1 + B\sqrt{v(B,\theta)^2 \cos^2\theta + u(B,\boldsymbol{\theta})^2 \sin^2\theta} / B_0)^{\beta} \quad (1)$$

Where I_{c0} is the self-field critical current, B_0 and β are constants that depend on the material. Coefficients u and v are functions of the applied field magnitude B and field angle θ.

$$u(B,\theta) = b(B)\cos\theta + c(B)\theta + d(B) \quad (2)$$

$$v(B,\theta)^2 = a(B)^2, \text{ When } \theta \geq 0$$

$$v(B,\theta)^2 = \left| f(B)(\theta - \theta_0)\exp(g(B)\theta) \right| \text{ ,When } \theta < 0 \quad (3)$$

Where a-d, f and g are the functions of the applied field magnitude B, and θ_0 is a constant, which again depends on the material.

For SP1, we find B_0 = 0.319, β = 2.405, I_{c0} = 101.4, and θ_0 = 5. Because of the asymmetric tape behaviour, the functions of a-d, f and g should be considered separately when $\theta \geq 0$ and $\theta < 0$.

For $\theta \geq 0$

$$a(B) = 0.7174\exp(-0.9624B) - 1.567\exp(-32.3B) \quad (4)$$

$$b(B) = -3.606\exp(-1.001B) + 5.353\exp(-12.93B) \quad (5)$$

$$c(B) = -3.509\exp(-0.981B) + 5.818\exp(-13.41B) \quad (6)$$

$$d(B) = 6.139\exp(-1.002B) - 8.715\exp(-13.96B) \quad (7)$$

For $\theta < 0$

$$b(B) = 5.087\exp(-1.372B) - 21.69\exp(-37.35B) \quad (8)$$

$$c(B) = -5.593\exp(-1.349B) + 20.09\exp(-34.71B) \quad (9)$$

$$d(B) = -9.557\exp(-1.366B) + 33.77\exp(-36.22B) \quad (10)$$

$$f(B) = 6.286\exp(-2.149B) - 15.94\exp(-29.13B) \quad (11)$$

$$g(B) = 8.19\exp(-1.81B) + 1.004\exp(1.519B) \quad (12)$$

For SP2, we find B_0 = 0.5, β = 1.446, I_{c0} = 94.7, and θ_0 = 2.7.

For $\theta \geq 0$

$$a(B) = 1.15\exp(-0.4108B) - 1.831\exp(-19.68B) \quad (13)$$

$$b(B) = -1.526\exp(1.625B) + 5.557\ln(B + 10^{-5}) + 28.27\exp(-9.14B) \quad (14)$$

$$c(B) = -8.954\exp(-0.4397B) + 15.33\exp(-10.36B) \quad (15)$$

$$d(B) = 15.61\exp(-0.4722B) - 23.38\exp(-10.77B) \quad (16)$$

For $\theta < 0$

$$b(B) = 4.997\exp(-0.596B) - 12.17\exp(-11.24B) \quad (17)$$

$$c(B) = -6.042\exp(-0.547B) + 13.88\exp(-12.63B) \quad (18)$$

$$d(B) = -10.97\exp(-0.5825B) + 22.62\exp(-13.62B) \quad (19)$$

$$g(B) = 4.046\sin(1.882B) + 54.94B\exp(-8.025B) \quad (20)$$

$$f(B) = 26.25\exp(-0.9123B) - 41.28\exp(-13.54B) \quad (21)$$

The two-variable direct interpolation method proposed here is a simpler and more direct method, similar to a look-up table . All of the experimental data can be input as a single function, with two input variables B and θ, and one output variable, the critical current density J_c, using a direct interpolation, which is available in Comsol. This significantly simplifies the process and improves the computational time.

Modelling and Simulation

$$\nabla \times \boldsymbol{E} + \mathrm{d}\boldsymbol{B}/\mathrm{d}t = \nabla \times \boldsymbol{E} + \mathrm{d}(\mu_0\mu_r\boldsymbol{H})/\mathrm{d}t = 0 \quad (22)$$

$$\nabla \times \boldsymbol{H} = \boldsymbol{J} \quad (23)$$

Where $\boldsymbol{H}$ represents the magnetic field strength components, $\boldsymbol{J}$ represents the current density and $\boldsymbol{E}$ represents the electric field. μ_0 is the permeability of free space, and for the superconducting layer and air, the relative permeability is simply μ_r = 1. In the 2D infinitely long model, $\boldsymbol{H}$ = [H_x, H_y], $\boldsymbol{J}$ = [J_z], $\boldsymbol{E}$ = [E_z].

$$\boldsymbol{E} = (E_0 / J_c(B,\theta))(|\boldsymbol{J}| / |J_c(B,\theta)|)^{n-1}\boldsymbol{J} \quad (24)$$

$$J_c(B,\theta) = I_c(B,\theta)/S \quad (25)$$

Where E_0 is the characteristic electric field 1 μV/cm and S is the cross-section of the superconducting layer. For HTS material, n is usually within the range of 5 (strong flux creep) and 50 (limiting value for HTS and LTS material). When n > 20, (24) becomes a good approximation of Bean's critical state model. Therefore, we assume n = 21.

The magnitude, B, and orientation, θ, of the magnetic field can be expressed by (26) and (27).

$$B = \sqrt{B_x^2 + B_y^2} \quad (26)$$

$$\theta = \arctan(B_y / B_x) \quad (27)$$

Where $B_{x,y} = \mu_0\mu_r H_{x,y}$

For the non-superconducting air sub-domain surrounding the superconducting layer, a linear Ohm's law is considered $\boldsymbol{E} = \rho\boldsymbol{J}$, where ρ is the specific, high constant resistivity for air.

Integral constraints are applied to represent the particular current flowing in superconducting layer. A transport current I_s through the cross-section $\boldsymbol{S}$ of the tape is

$$I_s = \int \boldsymbol{J} \cdot \mathrm{d}\boldsymbol{S} \quad (28)$$

The calculation of the ac loss [J/m/cycle] of the superconducting tape in the 2D infinitely long model

$$AC\ loss = \int_0^T \int \boldsymbol{E} \cdot \boldsymbol{J}\ dS\ dt \quad (29)$$

Where $\boldsymbol{T}$ is the period of one cycle

Acknowledgements

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Results and Discussion

In-Field DC Critical Current Calculation

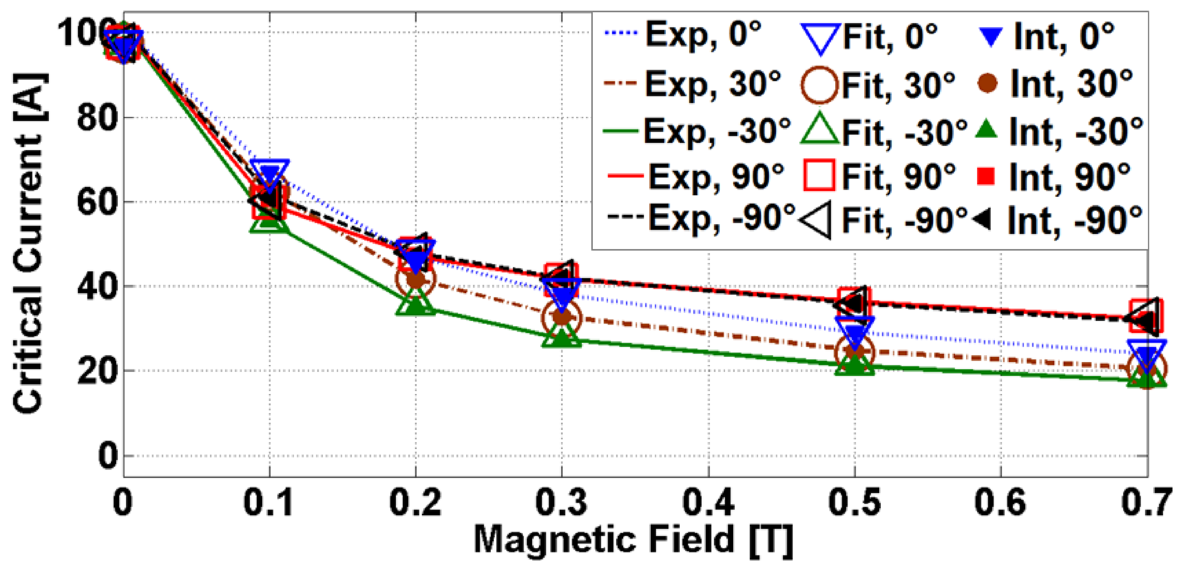

Fig. 5. Comparison of experimental results (lines), simulation results with data fitting (open symbols) and simulation results with two-variable interpolation (closed symbols) for short samples 1.

Table 1: Computational time required to calculate the in-field DC critical current using the data fitting and two –variable direct interpolation methods for short sample SP1 at different applied field angles. All values are given in units of seconds (s).

Applied Field	0°		-30°		30°	
	DF	INT	DF	INT	DF	INT
0 T	1958	344	2245	375	2145	368
0.1 T	36878	355	20180	383	15077	438
0.2 T	44864	339	28568	432	25593	395
0.3 T	53764	338	37963	397	33684	448
0.5 T	62247	316	44341	679	40437	629
0.7 T	71665	586	50764	590	48837	620

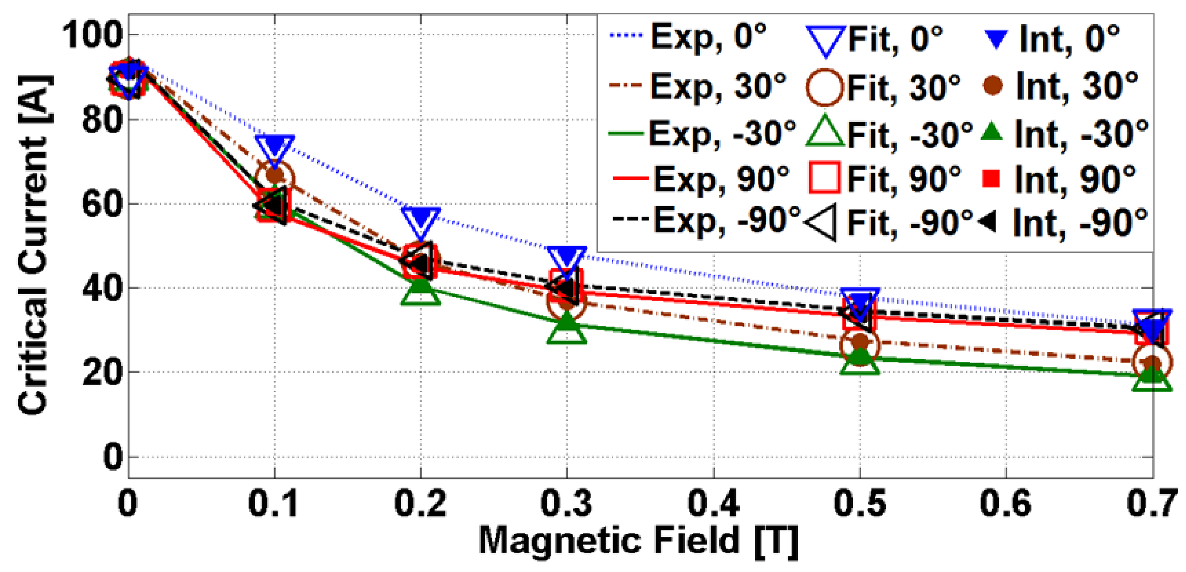

Fig. 6. Comparison of the experimental results (lines), simulation results with data fitting (open symbols) and simulation results with two-variable interpolation (closed symbols) for short samples 2.

Applied Field	0°		-30°		30°	
	DF	INT	DF	INT	DF	INT
0 T	1958	344	2245	375	2145	368
0.1 T	36878	355	20180	383	15077	438
0.2 T	44864	339	28568	432	25593	395
0.3 T	53764	338	37963	397	33684	448
0.5 T	62247	316	44341	679	40437	629
0.7 T	71665	586	50764	590	48837	620

Data fitting using the engineering formula and two variable direct interpolation are both accurate, but the two-variable direct interpolation is significantly faster than the data fitting method using an engineering formula.

AC loss calculation

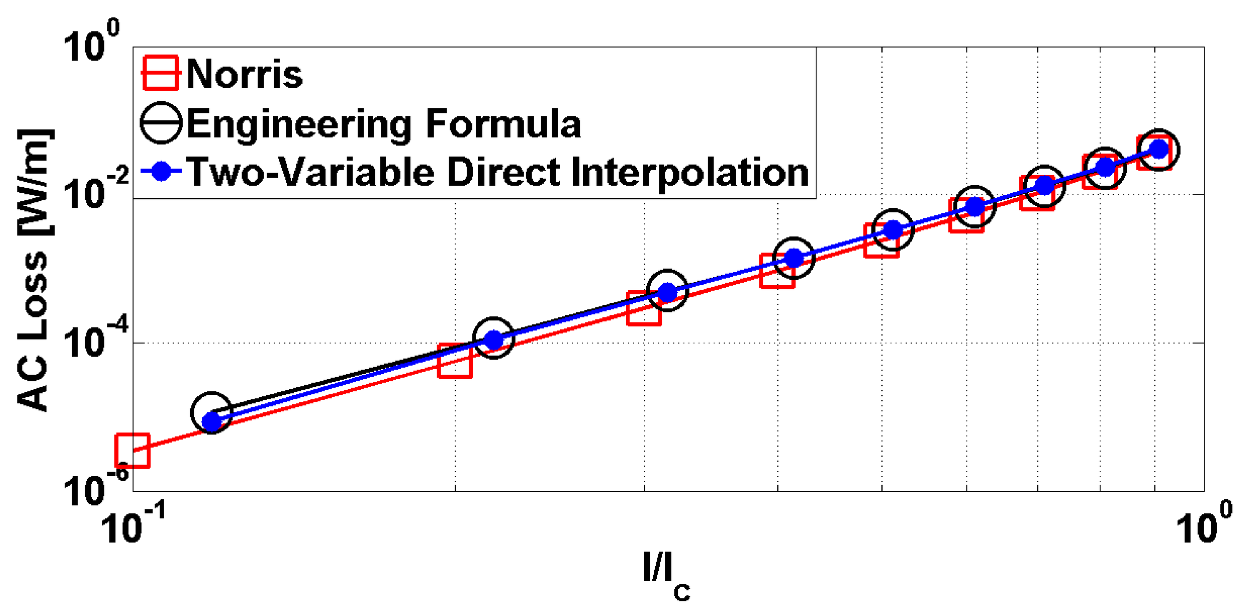

Fig. 7. Comparison of AC loss of samples 1 by three methods

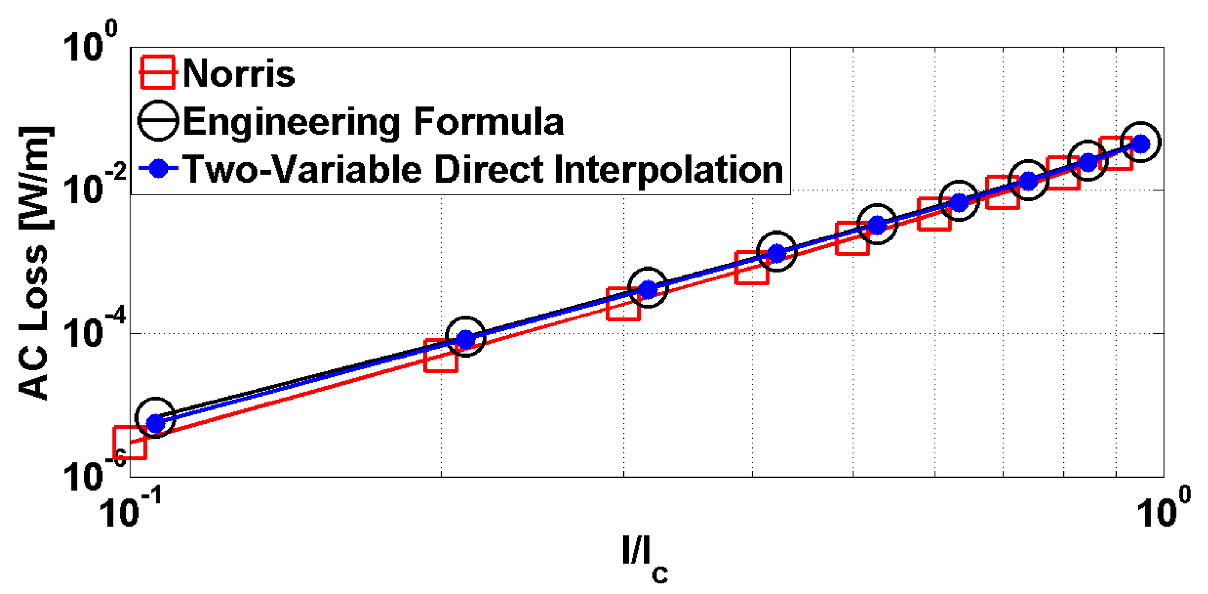

Fig. 8. Comparison of AC loss of samples 2 by three methods

AC loss calculation is consistent for data fitting using engineering formula and two-variable direct interpolation and consistent with the analytical solution.

In summary, the direct interpolation is recommended as the best method to include anisotropic $J_c(B,\theta)$ behavior to model HTS coated conductors in finite element models to achieve accurate, effective and efficient results.

Extension of Two-variable Direct Interpolation Method in 3D Model

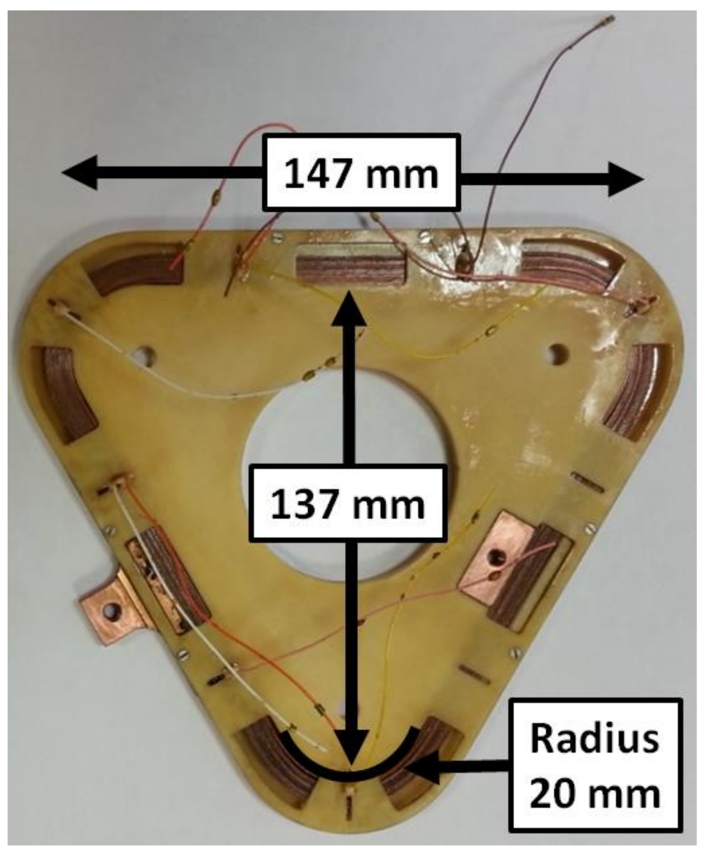

Fig. 9. Triangular HTS pancake coil

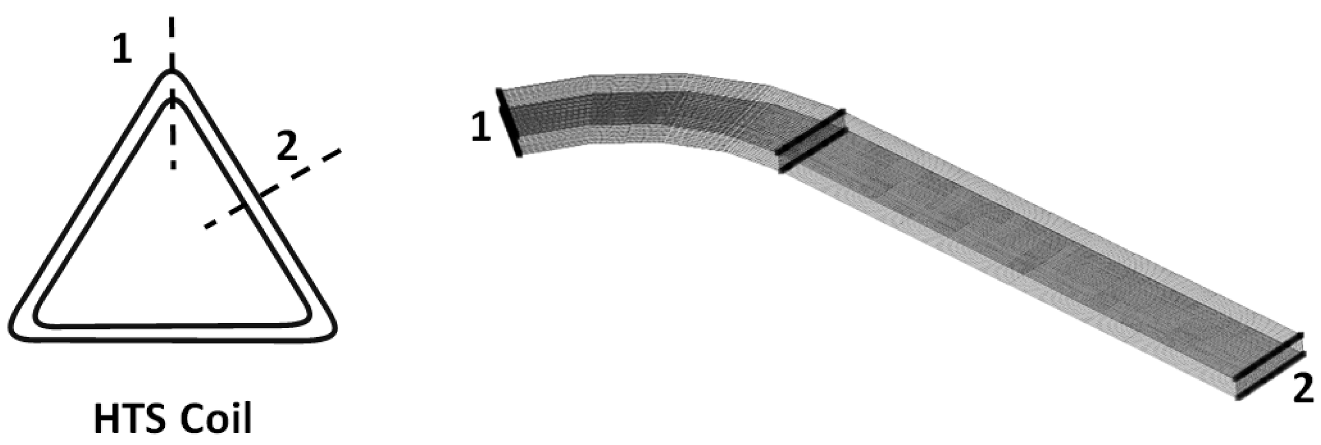

Fig. 10. 1/6th of the length of the coil is modelled using symmetric boundary conditions

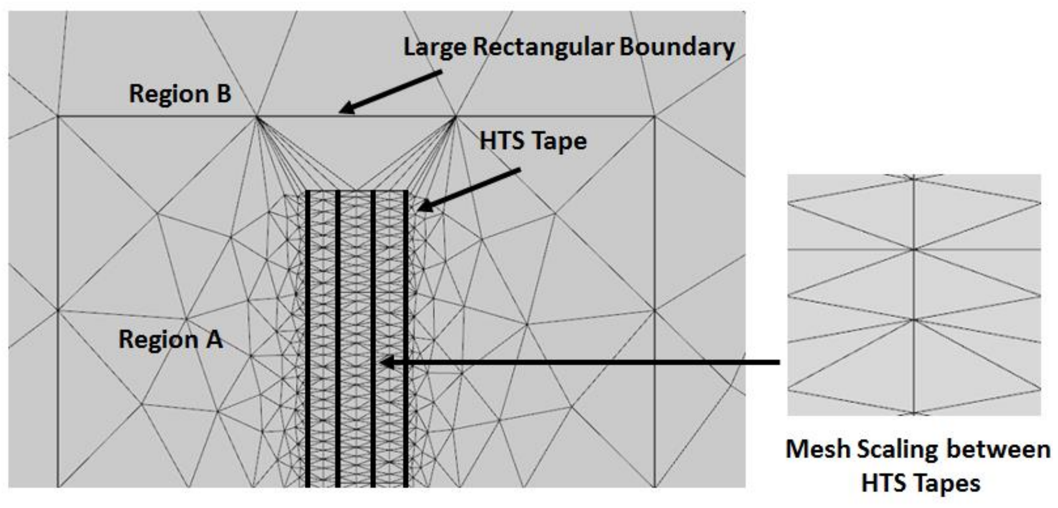

Fig. 11. Mesh example of the 2D cross-section of the 3D triangular coil

Two-variable direct interpolation is applied to include all of the $J_c(B, \Theta)$ data to avoid developing complicated equations for data fitting completely and greatly improve the computational speed. **A combined mapped-scaled** (Fig.5) is proposed to improve the convergence of 3D model and allow a fast and efficient 3D simulation include the real thickness of the superconducting layer. Two-variable direct interpolation is successfully applied in the 3D model.

In summary, the two-variable direct interpolation is an effective and efficient method, which can be applied widely in the superconducting model field. It is effective from 2D to 3D model.

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