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Theory of AC Loss in Cables with 2G HTS Wire

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Abstract. While considerable work has been done to understand AC losses in power cables made of first generation (1G) high temperature superconductor (HTS) wires, use of second generation (2G) HTS wires brings in some new considerations. The high critical current density of the HTS layer 2G wire reduces the surface superconductor hysteretic losses. Instead, gap and polygonal losses, flux transfer losses in imbalanced two layer cables and ferromagnetic losses for wires with NiW substrates constitute the principal contributions. Current imbalance and losses associated with the magnetic substrate can be minimized by orienting the substrates of the inner winding inward and the outer winding outward.

1. Introduction: Components of ac loss in 2G HTS cables

Garber *et al.*,[1] first developed the theory for the current distribution and ac loss in power cables helically wound with tape-shaped conductors around a cylindrical former. Their initial work was applied to cables based on Nb_3Sn. Later authors[2-5] addressed the theory of cables using first generation (1G) high temperature superconductor (HTS) wires based on BSCCO-2223.

Here we present the theory of current distribution and ac loss in such helically wound cables based on the second generation (2G) HTS wires based on thin films of YBCO-123. These wires introduce important new features compared to the earlier work on 1G cables, in particular the very thin HTS layer with its correspondingly high critical current density J_c and the ferromagnetism of wires using NiW substrates.

We focus on a relatively simple though still interesting case – the two-layer counter-wound cable, that is, a configuration in which the first layer consists of N_1 wires of width w helically wound with a pitch angle θ_1 and the second layer of N_2 wires of width w wound with a pitch angle θ_2 which is approximately the negative (opposite handedness) of θ_1. As is now well known, in such a configuration, currents to flow close to equally on both layers; this is often called a uniform current distribution (UCD) and is advantageous in reducing ac losses. This configuration is widely used in the industry as a phase conductor, although it is generally embedded in either a surrounding grounded screening layer or in concentrically wound multiple phases; these more complex configurations will be addressed in later work.

For a conceptual understanding, it is helpful to break down the ac loss mechanisms in such phase conductors into six types. The first can be called the surface Bean model loss because it is the same loss mechanism as Bean originally calculated on the surface of a superconducting slab in the presence of an ac magnetic field parallel to the surface;[6,7] this loss increases as the cubic power of the applied field and is a dominating contribution in 1G cables. In the simplest approximation, Vellego and Metra[2] introduced a "monoblock" model, in which the entire cable is considered as a tube of superconductor and these Bean model surface losses are calculated on the outside surface. In a more detailed model in which the helical structure of the multilayer cable is taken into account, but with non-conducting gaps of zero width between adjacent wires, such losses can be calculated on all the wire surfaces; this result is given in the next section. However, because the Bean model losses are inversely proportional to the superconductor critical current density J_c, and because J_c in 2G wires is two orders of magnitude larger than J_c in 1G wires, this term is generally negligible in 2G cables.

A second mechanism of ac loss in these helically wound phase conductors arises from finite gaps between adjacent wires, which perturb the azimuthal (circumferential) fields, generating perpendicular (radial) field components. The gap loss for a planar geometry was first calculated by Norris,[8] corrected later by Majoros;[9] it scales as the square of the gap and the fourth power of the applied field. The model has been extended to conforming wires around a cylindrical former by Mawatari.[10] A related third mechanism arises from the polygonal configuration of a cross section of the phase conductor if the wires do not conform to the cylindrical former.[11] The angle between the wires, even in the absence of a gap, perturbs the fields parallel to the surface and generates components perpendicular to the wire plane. The ac losses of these two mechanisms have recently been calculated for the case of a single layer of straight wires assembled around a cylindrical former; for typical parameters of 2G wires, their magnitude is significantly larger than the Bean surface losses.[10,11] In what follows we develop a theory assuming cylindrically conforming wires with non-conducting but infinitely narrow gaps between adjacent helical wires. We presume the gap and polygonal losses are additive to the additional loss mechanisms considered here.

The fourth mechanism can be called flux-cutting loss. It arises from the collision of flux fronts penetrating from the outer and inner surfaces, and having different flux orientations.[12] This is the least well understood of the loss mechanisms and will require further work to calculate.

The fifth mechanism arises when the current in a given winding layer (usually the outside layer) exceeds its total critical current I_{ci}. This can occur whenever current balance (UCD) is not achieved.[3] The transfer of current and flux which ensues can give rise to very large losses. A simplified treatment of flux transfer loss was first given by Daeumling in his cylindrical "duoblock" model.[5] To avoid this loss, we consider below the requirements for achieving current balance between the two helically wound layers, which are non-trivial particularly in the presence of a magnetic substrate. We also calculate this loss in several cases when current balance is not achieved.

The sixth mechanism arises from the presence of magnetic substrates in 2G HTS wire. The most widely used such substrate is Ni-5at%W, which has an extremely narrow and sharp hysteresis loop with a coercive field of only a few tenths of a mT, an initial susceptibility χ of 1800, and saturation at higher applied fields.[13] If the substrate is exposed to an ac magnetic field with an amplitude exceeding several tenths of a mT in the cable configuration, it will contribute a ferromagnetic (FM) loss per cycle corresponding to the full area of the MH hysteresis loop. At 60 Hz, this FM loss ranges from 3 to 7 mW/m for a 75 μm x 4 mm substrate, depending on the wire's mechanical treatment (cold working increases the coercive field).[14] As we shall see, it is difficult to avoid exposing the substrate to such fields, and so the FM loss corresponding to all the substrates in the cable phase conductor generally contributes to its total loss. This loss dominates at low currents and can reach 0.35 W/m for a 50-wire conductor. While this is modest compared to thermal and other losses in a typical HTS power cable, every loss reduction helps to reduce the refrigeration requirements; so textured substrates with even lower ac loss are being actively researched around the world.

Another mechanism for ac loss in the presence of magnetic substrates is the indirect influence of their stray fields on the flux penetration process in the superconductor. This effect may enhance – or potentially reduce – the gap losses depending on the exact configuration, e. g., whether the substrate extends beyond the HTS material or not. This is a complex phenomenon which is receiving initial attention, both analytically and numerically,[15,16] but we do not consider it further in this paper.

In what follows, we treat the magnetic material as a simple linear paramagnet with a dimensionless susceptibility $\chi = M/H$ [where $B = \mu_0(H + M) = \mu_0(1 + \chi)H$] , postponing till a future publication the more complex treatment of a full magnetic hysteretic loop. Nevertheless, the paramagnetic model already gives a strong indication of the most desirable configuration for the substrates in the phase conductor to preserve UCD and avoid large flux transfer losses.

2. Analysis of current balance in a two-layer cable with paramagnetic substrates

We follow the basic model introduced by Garber *et al.*,[1] applied to the two-layer cable, but now adding the flux due to magnetic substrates. We define R_i as the radius of the ith layer (layer 1 being the inner layer), I_i as the total time-dependent ac current in the ith layer, t_i as the tangent of the pitch angle θ_i of the ith layer (positive or negative corresponding to right or left-handed helices), and P_i as the corresponding pitch length; so $t_i = tan\theta_i = \pm 2\pi R_i/P_i$. Furthermore, d_i is the thickness of the paramagnetic substrate of the ith layer, the substrate being characterized by a dimensionless linear susceptibility $\chi=M/H$ and an index p_i which characterizes the presence and orientation of the magnetic substrates. In particular, four configurations of magnetic substrates can be identified by their orientation vis-à-vis the center of the cable: in-in (p_i = 1,1), in-out (1,0), out-in (0,1) and out-out (0,1). It turns out that the key difference between out-in and out-out is that out-in has twice as much magnetic material located between the two HTS layers; otherwise the theory for the current distribution is identical for the two cases. In the model, magnetic layers outside the two HTS layers do not affect the current distribution.

From Ampere's law, one first evaluates in terms of I_1 and I_2 the axial and azimuthal magnetic fields H within, between and outside the two layers. Next one calculates the flux including the contribution of the magnetic material, and finally the voltages per length induced in the two layers by the rate of flux change according to Faraday's law. Resistance in the wires and terminations is assumed to be zero, which is valid for currents not exceeding the layer critical currents. This leads to the net voltages V_i per unit length of the two layers:

$$V_1 = L_{11}dI_1/dt + L_{12}dI_2/dt \quad , \qquad (1)$$
$$V_2 = L_{21}dI_1/dt + L_{22}dI_2/dt \quad , \qquad (2)$$

where

$$L_{11} = (\mu_0/4\pi)[t_1^2(1 + 2p_1d_1\chi/R_1) + 2ln(R_2/R_1) + 2p_2d_2\chi/R_2] \quad , \qquad (3)$$
$$L_{12} = L_{21} = (\mu_0/4\pi)(R_1/R_2)t_1t_2(1 + 2p_1d_1\chi/R_1) \quad , \qquad (4)$$
$$L_{22} = (\mu_0/4\pi)t_2^2(1 + 2p_1R_1d_1\chi/R_2^2 + 2p_2d_2\chi/R_2] \qquad (5)$$

are the components of the matrix of inductances per unit length.

Equating the two voltages, one can find the current ratio

$$I_1/I_2 = (L_{22} - L_{12})/(L_{11} - L_{21}) \ . \qquad (6)$$

Some insight into the fields and current flows in such cables can be obtained from the simple nonmagnetic case with $\chi = 0$, and with $R_1 \approx R_2 = R$ and $\theta_2 = -\theta_1 = \theta$. The resulting current flows and fields are illustrated schematically in figure 1, where the two layers of the cylindrical cable structure are cut along their length and laid flat. In this case the net currents in the two layers are equal; so the axial field in the center of cable is zero, which means that no surface current flows on the inner surface of the inner layer of windings. Therefore the outer surface of that layer must carry all the current $I_1 = I/2$, which corresponds to a sheet current $I/2Nw = I/4\pi Rcos\theta$, indicated by the solid arrow on the left of figure 1.

According to Ampere's law, the magnetic field at the upper surface of the first (inner) layer equals the sheet current in magnitude and is oriented perpendicular to it, as shown schematically in figure 1. This highlights the fact that in such a helically wound cable, the magnetic field in the intermediate region between the two winding layers is substantial. Therefore, if Ni-W magnetic substrates are present in this intermediate region between the two HTS winding layers, they will be magnetized and contribute FM loss. Compare this to the case of two straight parallel HTS tapes on top of each other with magnetic substrates oriented inward towards each other.[17] When currents flow equally down those two tapes, the intermediate region between the two HTS layers is screened, so that the magnetic material in the intermediate region experiences near zero magnetic field. As a result, the magnetic material contributes no FM loss in an ac current, as beautifully confirmed in experiment.[17] Thus, facing magnetic substrates towards each other can help reduce ac loss in the case of straight parallel wires, but not in the case of the two layer helically wound cable.

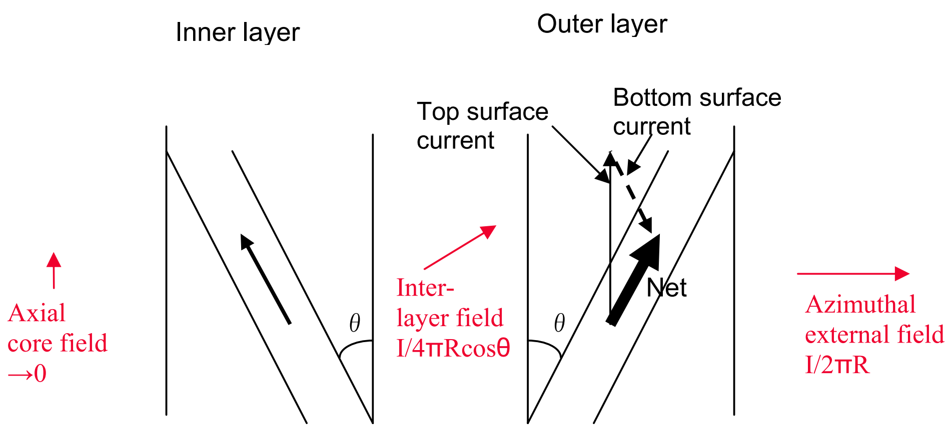

Figure 1. Schematic current flows and fields in two-layer helical counterwound cable with layers laid flat and $R_2 = R_1$.

Since the outer HTS layer in figure 1 must screen the intermediate field, a surface screening current must arise on the inner surface of the outer layer which is, in the limit of equal radii, equal and opposite to the surface current on the outer surface of the inner layer. This sheet current on the inner surface of the outer layer is indicated as a dashed arrow. As is evident in figure 1, such a sheet current is not parallel to the wire; it intersects the edges of the wires in this layer. It must then travel around the edge to the outer surface, where another sheet current must flow, with magnitude $I/2\pi R$ and oriented down the axis of the cable, as indicated in the solid arrow. By simple geometry, it is evident that such a surface current on the outer surface, in combination with the surface current on the lower surface, forms a net current exactly along the direction of the wires of this layer. In other words, the current flow *within the HTS layer* of the outer winding forms a remarkable helical pattern, as first recognized by Garber *et al.*[1]

Since the surface Bean hysteretic loss per cycle per unit area is[6,7] $2\mu_0 h_0^3/3J_c$, where h_0 is the ac field amplitude parallel to the surface, the losses of the three active surfaces can be combined to give the net Bean surface loss Q_s per cycle per unit length for the two layer cable with $R_1 = R_2 = R$ and $\theta_2 = -\theta_1 = \theta$:

$$Q_s\ (joules/m\text{-}cycle) = (d_s/6\pi R)\ \mu_0 I_c^2\ [(1 + 4\cos^3\theta)/\cos^2\theta]\ F^3\ , \qquad (7)$$

Where d_s is the HTS layer thickness, $F = I/I_c$ and where I and I_c refer to the currents of the entire cable. This loss scales as $d_s F^3$, the HTS layer thickness times the cube of the normalized current, and so, except at very small F and for typical values of the lateral gap g, it is typically small compared to gap losses[8,11] which scale as g^2F^4/w. Equation (7) holds up to $F_x = 1/(1 + 2\cos\theta)$, above which point the flux fronts from the two surfaces of the outer layer intersect, undergoing what has been termed a flux-cutting process.[11] As mentioned above, the ac loss in this region has not yet been calculated.

Next let us consider the nonmagnetic case where the radii and pitch angle magnitudes are close to each other but not equal; then to first order in $\Delta i = (I_2-I_1)/I_{ave}$, $\Delta p = (P_2-P_1)/P$ and $\Delta r = (R_2-R_1)/R$ (where I_{ave}, P and R are the average values of both layers), equation (6) reduces to

$$\Delta i = \Delta p + \Delta r[(P/2\pi R)^2 - 1]\ . \qquad (8)$$

Thus the currents in this case can be balanced either by using equal pitch lengths in the two layers and $P = 2\pi R$ corresponding to a pitch angle $\theta = 45$ deg, or by adjusting the pitch lengths to compensate for the second term in equation (8) when $\theta \neq 45$ deg. Note that to enable cable bending flexibility in practice, the pitch angle must be significantly less than 45 deg, in which case current balance ($\Delta i = 0$) requires adjustment of pitch lengths.

When the pitch lengths of the two layers are equal, the axial field in the center of the cable can be calculated to first order:

$$H_{core,axial} = (I/P)\Delta r[(P/2\pi R)^2 - 1] \ . \tag{9}$$

Unless $\theta = 45$ deg, this field is always finite and for typical values is in the range of several mT, larger than the coercive field of Ni-5W.

By making the pitch lengths unequal, it is possible in principle to make the axial field $H_{core,axial}$ zero. Since

$$H_{core,axial} = t_1 I_1/2\pi R_1 + t_2 I_2/2\pi R_2, \tag{10}$$

and $V_1 = V_2$ [equations (1) and (2)], in order to achieve $H_{core,axial} = 0$ we must have

$$t_1 t_2 = -\frac{(R_1/R_2)\ln(R_2^2/R_1^2)}{1 - R_1^2/R_2^2} \ . \tag{11}$$

When R_1 is very close to R_2, the right-hand side is very nearly equal to -1, and $\theta_2 = -\theta_1 = 45$ deg is a solution. However, if $\theta_2 < 45$ deg, we must have $|\theta_1| > 45$ deg. Since practical values of the magnitude of the pitch angle must be well below 45 deg, it is not possible to achieve $H_{core,axial} = 0$ in practice. An important consequence of this is that the FM loss of magnetic substrates in the cable core is difficult to avoid.

Now let us consider the paramagnetic case. The current ratio in the two layers is calculated from equation (6); results are shown in figure 2 as a function of χ for a cable with 20 wires of width 4.4 mm on the two layers, with an inner layer of radius of 15.0 mm wound as a left-handed helix with a pitch angle $\theta_1 = -20.97$ deg , and an outer layer of radius 15.5 mm wound as a right-handed helix with a pitch angle $\theta_2 = 25.37$ deg. The substrate thickness is $d_1 = d_2 = 75$ µm. Results for the four possible combinations of substrate orientations are shown: in-out, in-in, out-out and out-in. The in-out configuration, in which magnetic substrates are excluded from the intermediate region between the winding layers, shows by far the least effect of χ on the current balance. A simple physical way to understand this result as follows: magnetic substrates lying outside the outer superconductor layer cannot affect the current distribution between the two layers because in our model they do not couple inductively to either of the layers. By contrast, the rate of change of the flux arising from any magnetic substrate lying in the intermediate region between the two HTS layers will induce a loop current around the two layers and through the terminations at the ends, thus imbalancing the net currents on the two layers.

Note that this result differs remarkably from ac loss results for two straight wires oriented with their magnetic substrates facing each other[17]; in this latter case the surrounding HTS layers screen the magnetic substrates, and so the ferromagnetic ac loss contribution disappears if the layers are close enough to each other. Unfortunately, in helical windings, the symmetry of the straight-wire case is lost, so that a field appears between the two HTS layers and the ferromagnetic loss is hard to avoid. Because of the flux-transfer losses to be discussed below, the most favourable case for reducing ac loss in helical windings is with HTS layers facing each other (the in-out configuration), rather than with magnetic substrates facing each other.

Although the linear paramagnetic model differs from the actual non-linear susceptibility of Ni-5W, these results nevertheless give a strong indication that current imbalance in phase conductors using wires with magnetic substrates can be minimized using the substrate in-out configuration. This is a key conclusion of this paper. While the substrates still experience magnetic fields and so contribute FM loss, this loss is modest compared to the loss which can arise in the other configurations, as we will show below.

Another interesting result is obtained by calculating the core axial field to first order in $1/\chi$:

$$H_{core,axial} = t_2[2R_1R_2 ln(R_2/R_1) + (R_2^2 - R_1^2)t_1t_2]I/4\pi d_1(R_2t_1 - R_1t_2)^2\chi \ , \tag{12}$$

which goes to zero as $\chi \rightarrow \infty$, while the magnetization $M_{core,\ axial} = \chi H_{core,axial}$ remains finite.

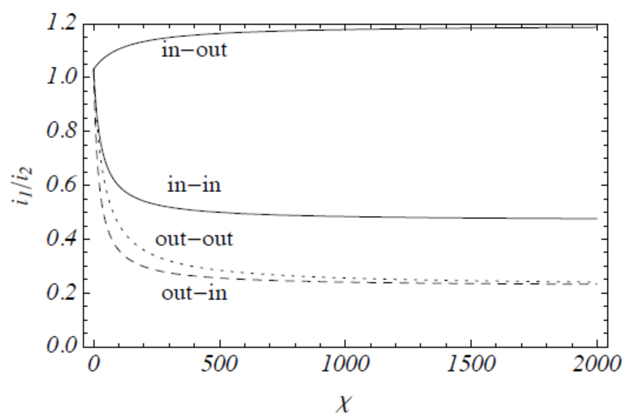

Figure 2. Ratio of currents in two layers of counter-wound helical cable, for different orientations of substrate of susceptibility $\chi = M/H$. For example, "in-out" means substrate of inner layer 1 faces in towards the cable core, while substrate of outer layer 2 faces out. Parameters are given in the text.

Imbalanced currents (for example, $i_1/i_2<1$) give rise to major flux-transfer loss when the current in a given layer exceeds its total critical current. In this case current is forced to transfer from the outer to the inner layer, which is accompanied by flux transfer in the region Δr between the two layers. The details of the calculation will be given elsewhere; the result is

$$Q_{ft}(joules/m\text{-}cycle) = 4\ (L_{11} - L_{12})I_cI_{c2}(F - F_{ft})\ , \tag{13}$$

where

$$F_{ft} = I_{peak,ft}/I_c = [1+(I_1/I_2)]I_{c2}/I_c\ , \tag{14}$$

and I_1/I_2 is given by equation (6). For example, consider a counterwound cable with $t_1 = -t_2,\ R_1 \approx R_2,\ I_{c2} = I_c/2$, in the limit of large χ. With an in-out configuration, both I_1/I_2 and F_{ft} are close to 1; hence no flux transfer loss occurs throughout nearly the entire range of applied current. However, for an in-out configuration and the same parameters, $I_1/I_2 = t_2^2$; so for example, at $\theta = 20$ deg, $F_{ft} = 0.56$ ($I_{rms}/I_c = 0.40$), above which the flux transfer loss from equation 13 is

$$Q_{ft} = (\mu_0/\pi R_2)\chi d_2 I_c^2(F - F_{ft}). \tag{15}$$

This loss is dominated by the magnetic material between the two HTS layers and diverges with χ. For a frequency f = 60 Hz, I_c = 5000 A, d_2 = 150 μm (for an out-in configuration with two 75 μm thick substrates), and even a very modest χ = 10, one finds a very substantial loss of $Q_{ft}f = 60\ (F - F_{ft})$ watts/m. This estimate highlights the importance of using the in-out configuration to avoid large imbalance between the two winding layers and consequent large flux transfer losses at intermediate current levels.

In conclusion, using a model in which gaps between adjacent HTS wires are assumed infinitely small, we have derived formulas which predict current flow in cable phase conductors with the two layers of HTS wires wound in a helical configuration. We have treated magnetic substrates in the paramagnetic limit. Different orientations of the magnetic substrates are found to give very different current flows in the two layers, and the "in-out" configuration (substrate of inner layer facing inward, and substrate of outer layer facing outward) is the most effective to maintain current balance on the two layers. This is important to avoid substantial flux transfer losses, which we have also calculated in this model. As long as flux-transfer losses can be avoided – by using the counter-wound two-layer configuration with in-out substrates, the FM loss of typical Ni-5W magnetic substrates is modest. In future work, more complex configurations used in actual power cables will need to be addressed, including the shielded single-phase and coaxial three-phase configurations, and also a more realistic hysteretic model for the magnetic substrate.

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