Quantum Breathers in Coupled Josephson Junctions

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***Abstract* – Anharmonicity present in the potentials of two capacitively coupled Josephson junctions allows for quantum excitations that localize energy on one junction during a time that sensitively depends on the excitation energy, and can be tuned through the bias current injected into the junctions. Manipulation techniques that nowadays are used for quantum information processing with Josephson junctions can be used to resolve the flow of energy between the junctions in time.**

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I. INTRODUCTION

The fact that Josephson junctions (JJ) are nonlinear devices with macroscopic quantum behavior has made them serious candidates for practical applications in quantum information processing, where the first two energy levels of the junction are used as quantum bits (qubits). Several techniques for manipulating these devices in the quantum regime have been already developed [1, 2].

Recently, we have shown that these techniques may be used for studying the evolution of excitations [3], where energy is localized on one of the junctions for a very long time due to the anharmonicity in the potentials. We considered a system consisting of two capacitively coupled Josephson junctions. We analyzed its spectral properties seeking for quantum breathers (QB), which in this case are pairs of nearly degenerate eigenstates. By highly exciting only one JJ, a strong overlap with QB states induces a slow tunneling between both junctions. The tunneling time sensitively depends on the overlapping QB pair, and therefore on the initial level of excitation.

II. ENERGY SPECTRUM AND EIGENSTATES

The system is sketched in Fig. 1a [4]: Two JJs with capacitance C_J and critical current I_c are interacting via a capacitor C_c with coupling strength $\zeta = C_c / (C_c + C_J)$. The bias current injected into the junctions is I_b. The Hamiltonian of the system is

$$H = H_1 + H_2 + \zeta V \,, \qquad (1)$$

where $H_{1,2}$ is the Hamiltonian of each junction that includes the washboard potential $U(\varphi_{1,2}; I_b)$ sketched in Fig. 1b, and $\varphi_{1,2}$ are the Josephson phase differences. $V = V(P_1, P_2)$ is the interaction that couples the junctions through the conjugate momenta $P_{1,2}$.

When the junctions are in the superconducting state, the system behaves as two coupled anharmonic oscillators. For the classical dynamics, the equations of motion

admit discrete breather solutions [5], which are time periodic and for which the energy is localized predominantly on one of the junctions (see Fig. 1c).

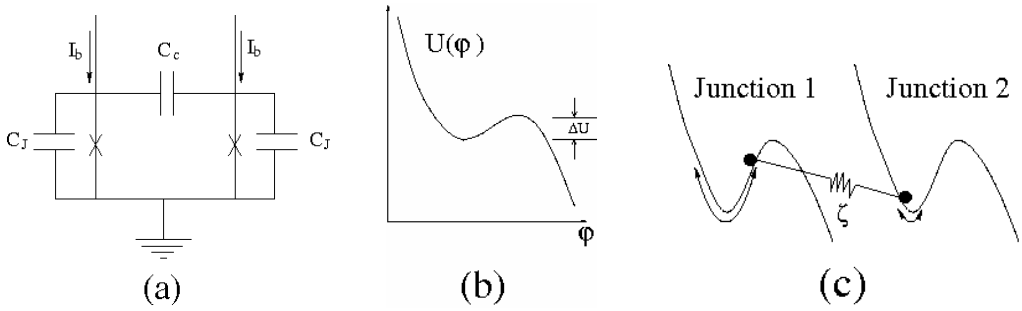

Fig. 1. (a) Circuit diagram of the capacitively coupled JJs. (b) Sketch of the washboard potential of a single current-biased JJ. (c) Sketch of a discrete breather solution in the classical dynamics of the system.

For the quantum dynamics we solve the Schrödinger equation and find the energy spectrum and eigenstates of the Hamiltonian (1) written in the basis of product states $\{|n_1, n_2\rangle = |n_1\rangle \otimes |n_2\rangle\}$. Here, $|n_i\rangle$ is an eigenstate of both the single-junction Hamiltonian operator $\hat{H}_i$ and the corresponding number operator $\hat{n}_i$ with eigenvalue n_i, that counts the energy levels inside the potential well of the junction. We neglect quantum escape from the potential well by imposing a hard wall at the top of its barrier [3]. This is justified by a final comparison of the obtained tunneling times with the true state-dependent escape times. Due to the permutation invariance of the Hamiltonian all eigenstates are symmetric or antisymmetric.

To identify QB states whose corresponding classical orbits localize energy, we define the correlation functions

$$f_\mu(i,j) = \langle \hat{n}_i \hat{n}_j \rangle_\mu = \langle \chi_\mu | \hat{n}_i \hat{n}_j | \chi_\mu \rangle, \qquad i, j = 1, 2, \qquad (3)$$

where $\{|\chi_\mu\rangle\}$ is the set of eigenstates of the system. The ratio $0 \le f_\mu(1,2) / f_\mu(1,1) \le 1$ measures the site correlation of quanta in the junctions: it is small when quanta are site correlated (when there are many quanta on one junction there are almost none on the other one and vise versa) and it is close to unity otherwise.

In Fig. 2 we plot the nearest-neighbor energy spacing (splitting) and the correlation function of the eigenstates. For this and all the rest we use $I_c = 13.3$ μA, $C_J = 4.3$ pF, and $\zeta = 0.1$, which are typical values in experiments. We see that in the central part of the spectrum the energy splitting becomes small in comparison to the average, and the corresponding pairs of eigenstates (QBs) are site correlated. When these states are excited, many quanta are localized on one junction and the tunneling time of such an excitation from one junction to the other (given by the inverse energy splitting between the eigenstates of the pair) can be exponentially large.

III. TIME EVOLUTION OF SINGLE SITE EXCITATIONS

We compute the time evolution of different single site excitations, and the expectation values of the number of quanta at each junction $\langle n_i \rangle(t) = \langle \Psi(t) | \hat{n}_i | \Psi(t) \rangle$. Results are shown in Fig. 3. The initial excitation $|\Psi_0\rangle = |0,19\rangle$ (fig. 3b) overlaps strongly with QBs in the central part of the spectrum, which are site correlated and nearly

degenerate (arrow 2 in fig. 2b). The tunneling time of such an excitation is very long, and thus keeps the quanta localized on the initially excited junction for the finite time of observation. On the other hand, the initial excitations $|\Psi_0\rangle = |0,5\rangle$ and $|\Psi_0\rangle = |9,19\rangle$ (Figs. 3a and c) do not overlap with QBs (arrows 1 and 3 in fig. 2b), hence they tunnel after a short time.

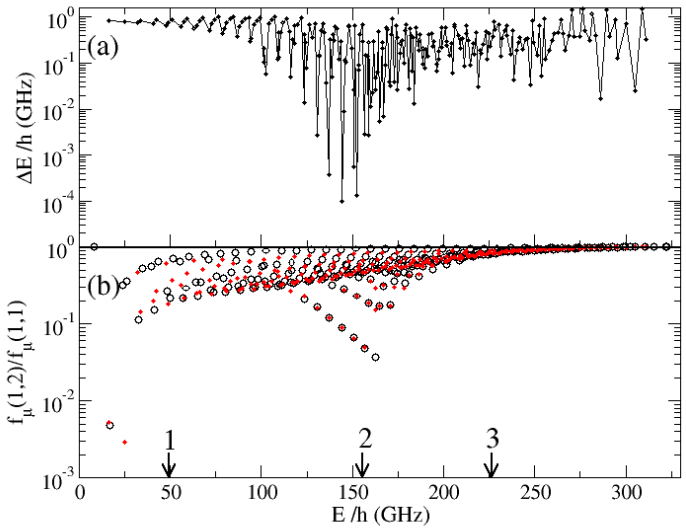

Fig. 2. (a) Energy splitting and (b) correlation function vs. energy of the two-junctions system (open circles, symmetric eigenstates; filled circles, antisymmetric eigenstates). The labeled arrows mark the energy corresponding to the eigenstates with strongest overlap with the initial excitations in Fig. 3 (see text). The parameters are $I_b / I_c = 0.945$ and $\zeta = 0.1$ (22 levels per junction).

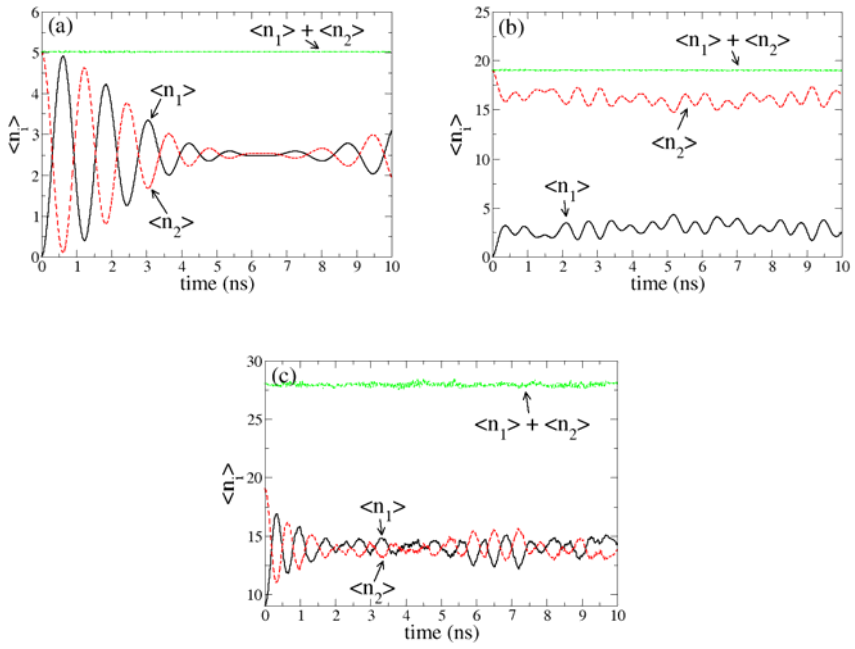

Fig. 3. Time evolution of expectation values of the number of quanta at each junction for different initial excitations. (a) $|\Psi_0\rangle = |0,5\rangle$; (b) $|\Psi_0\rangle = |0,19\rangle$; (c) $|\Psi_0\rangle = |9,19\rangle$. The parameters are $I_b / I_c = 0.945$ and $\zeta = 0.1$ (22 levels per junction).

IV. PROPOSED EXPERIMENT TO OBSERVE QBs

To experimentally observe the time evolution of excitations that strongly overlap with QBs, one could use the scheme of McDermott *et al* for simultaneous state measurement of coupled Josephson phase qubits [6]. After applying a microwave pulse to initially excite one of the junctions, their populations at a later time could be measured by applying simultaneous microwaves pulses to both of them and test which junction switches to the resistive state. Even though we neglected quantum escape from the potential wells of the junctions, it would not prevent us from observing the slow tunneling motion described in the last section (Fig. 3b), because the escape times are large enough [3]. As for decoherence (not taken into account in our model), an estimation gives a coherence time between 3 ns and 6 ns for junctions with high quality factors [7], which is much smaller than the tunneling time of excitations involving QB states. Therefore one would expect to see, instead of tunneling, a freezing of the excitation on one of the junctions before they decohere due to relaxation. Perhaps one could obtain more feasible results by increasing the bias current in such a way that there are less energy levels in the junctions.

V. CONCLUSIONS

It was shown that in a system of two capacitively coupled JJs one can excite QBs by highly exciting one of the junctions. As a result, the single-site excitation tunnels very slowly to the other junction. Whether a single-site excitation has strong or weak overlap with QBs, depends sensitively on its energy. Tuning that energy into the domain of strongest overlap with QBs, we may vary the tunneling times by several orders of magnitude. This could be experimentally observed by using the available techniques for manipulating JJ qubits, despite the existence of quantum escape in the junctions. We expect that further improvements in preparation and isolation of JJs will lead to long enough coherence times to allow the observation of the described phenomena.

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