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An Approach to Optimization of the Superconducting Quantum Interference Device Bootstrap Circuit

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Abstract – **Recently, we demonstrated and analyzed the superconducting quantum interference device (SQUID) bootstrap circuit (SBC). It is a direct readout scheme for dc SQUID in the voltage bias mode permitting one to suppress the preamplifier noise. The SBC enables us to control the two key parameters of a voltage-biased SQUID: the flux-to-current transfer coefficient and the dynamic resistance. The flux-to-current, $I - \Phi$, characteristics of SBC is made asymmetric by introducing the additional current feedback. Depending upon the choice of the working point, this feedback can be positive (working point W_2 on steeper $I - \Phi$ slope) or negative (W_1 on the less steep slope). The dynamic resistance is controlled by the additional voltage feedback. In our publications to date we presented only the SBC operation at W_2, while in this paper we demonstrate operation at W_1 and show that also in this regime the preamplifier noise suppression is possible. We used a liquid-helium-cooled Nb SQUID with loop inductance of 350 pH and attained white flux noise of 2.5 $\mu\Phi_0/\sqrt{Hz}$ both at W_2 and at W_1. In the latter case, the linear flux range exceeded one half flux quantum Φ_0. This large linear range should lead to a significantly improved stability and slew rate of the system and also make the tolerable spread in circuit parameters much wider than in all SQUID direct readout schemes known to date. Consequently, operation in this regime opens a new path to possible SBC optimization.**

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I. INTRODUCTION

The output signal of a direct-current superconducting quantum interference device (dc SQUID) is usually read out by a subsequent room-temperature electronic stage. In order to reduce the preamplifier noise contribution, the external flux modulation (FM) scheme is used as the standard readout technique [1]. A transformer at the input of such electronics is used to improve the impedance matching between the SQUID and the preamplifier. However, simpler and less expensive readout electronics is always preferred, and this is especially true in multichannel system applications. In the 1990s, two direct readout schemes involving neither FM nor a transformer were developed, namely the Additional Positive Feedback (APF) first proposed by Drung [2] and the Noise Cancellation (NC) scheme of Kiviranta and Seppä [3]. They used similar passive circuits, each parallel to the SQUID and consisting of a resistor R (or a FET as a variable resistor) in series with a feedback coil of inductance L_f coupled inductively to the SQUID loop, but operated these in different SQUID bias modes. In the current bias mode, the APF results in asymmetric flux-to-voltage (V - Φ) characteristics of the SQUID. The flux-to-voltage transfer coefficient, $\partial V/\partial \Phi$, is increased when the working point is set on the steep slope of such V - Φ characteristic. The preamplifier noise contribution is thus reduced. The NC technique uses voltage bias mode and the additional feedback doesn't affect the shape of the flux-to-current (I - Φ) characteristic which remains symmetric. However, on its two slopes (ascending and descending) the preamplifier noise contributions are different. On one slope, the noise is suppressed, while on the other it is increased.

Recently we demonstrated a voltage-biased SQUID direct readout scheme, which we named the SQUID bootstrap circuit or SBC [4]. In addition to the *R-L* feedback branch parallel to the SQUID, an additional current feedback is provided by a coil in series with the SQUID and coupled to its loop. A similar additional current feedback was already used by Drung, but he examined it only in the current bias mode [5]. In Figure 1 we show the schematic diagram of our circuit, including the two preamplifier noise sources, voltage noise V_n and current noise I_n. In this voltage-biased SQUID, the current feedback via the mutual inductance M_1 results in an asymmetric I – Φ characteristic with flux-to-current transfer coefficient, $\partial I/\partial \Phi$, increased on the steeper slope, while the parallel branch R_s-L_2 allows us to independently control the dynamic resistance $R_{d(SBC)}$ of this scheme and thus the $\partial V/\partial \Phi_{(SBC)} = \partial I/\partial \Phi_{(SBC)} \cdot R_{d(SBC)}$.

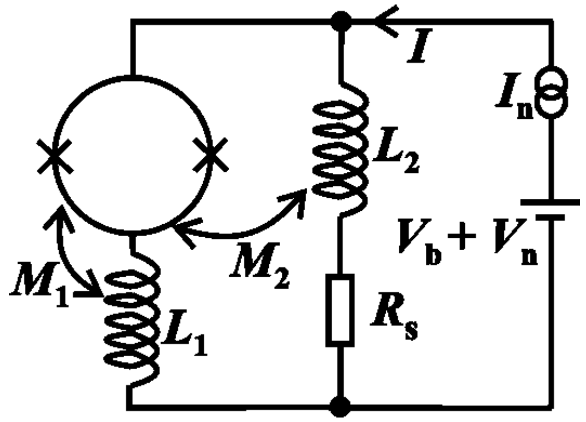

Fig. 1. Schematic diagram of the SBC scheme with preamplifier equivalent noise sources.

In our studies of SBC reported to date [4-6] we always placed the working point W_2 on the steeper I - Φ slope to enhance $\partial I/\partial \Phi_{(SBC)}$ and showed that when $\partial V/\partial \Phi_{(SBC)}$ reaches 1 mV/Φ_o the preamplifier noise contribution is suppressed to below the intrinsic SQUID noise. We also showed that the SBC parameter adjustment margins are then significantly wider than in either APF or NC schemes. In this paper, we examine the SBC performance when the working point W_1 is on the less steep I - Φ slope and show that this opens an alternative path to performance optimization.

II. BASIC CONSIDERATIONS

Any dc SQUID can be considered as a flux-to-voltage ($\partial V/\partial\Phi$) converter and a nonlinear resistor R_d (the dynamic resistance of the SQUID) which are connected in series. If a constant bias voltage V_b is applied across the dc SQUID, an external magnetic flux Φ_e results in a current change through the SQUID determined by $(\partial I/\partial\Phi) = (\partial V/\partial\Phi)/R_d$. Two noise sources from the preamplifier, V_n and I_n, should be taken into account in SQUID readout schemes. At the input of a voltage-biased SQUID, the equivalent flux noise of the preamplifier, $\delta\Phi_{preamp}$, can be generally expressed as:

$$\delta\Phi_{preamp.} = \sqrt{(\delta\Phi_{V_n})^2 + (\delta\Phi_{I_n})^2} = \frac{\sqrt{(V_n/R_d)^2 + {I_n}^2}}{\partial i/\partial\Phi} \qquad (1),$$

in which $\delta\Phi_{Vn}$ and $\delta\Phi_{In}$ are the equivalent flux noise components caused by V_n and I_n. We used equation (1) to calculate $\delta\Phi_{preamp.}$ of the preamplifier AD797 used in our experiments. From its data we calculated $\delta\Phi_{preamp.}(f)$ plots shown in Figure 2 for two $\partial I/\partial\Phi$ values, 10 $\mu A/\Phi_0$ and 5 $\mu A/\Phi_0$ (left and right axes).The $\delta\Phi_{preamp.}(f)$ is consistently dominated by V_n rather than I_n, also for very high R_d values[1]. The $\delta\Phi_{preamp.}$ decreases with increasing R_d and $\partial I/\partial\Phi$, *i.e.*, with $\partial V/\partial\Phi$. When $\partial V/\partial\Phi \geq 1$ mV/Φ_0, the AD797 value of $\delta\Phi_{preamp.}$ reduces to ≤ 1 $\mu\Phi_0/\sqrt{\text{Hz}}$, which is lower than the intrinsic noise of most SQUIDs with a loop-inductance L_s of ≥ 200 pH. As shown in Figure 2, for $\partial V/\partial\Phi = 1$ mV/Φ_0, R_d should reach 100 Ω with $\partial I/\partial\Phi = 10$ $\mu A/\Phi_0$ and 200 Ω with $\partial I/\partial\Phi = 5$ $\mu A/\Phi_0$.

Our SBC offers the possibility to independently set $R_{d(SBC)}$ and $\partial I/\partial\Phi_{(SBC)}$for suppressing the preamplifier noise to below the SQUID intrinsic noise . Therefore, we should be able to successfully operate our SBC also when the working point W_1 is on the less steep I - Φ slope. To do that it should suffice to wind coils L_1 and L_2 in the same rather than opposite directions and choose a sufficiently high dynamic resistance $R_{d(SBC)}$. According to the circuit analysis, $R_{d(SBC)}$ at working points W_2 and W_1 is:

$$R_{d(SBC)}^{W_2} = \frac{R_s\cdot[R_d - |M_1|\cdot(\partial V/\partial\Phi)]}{R_s + R_d - (|M_2| + |M_1|)\cdot(\partial V/\partial\Phi)} \qquad (2)\ [6]$$

$$R_{d(SBC)}^{W_1} = \frac{R_s\cdot[R_d + |M_1|\cdot(\partial V/\partial\Phi)]}{R_s + R_d - (|M_2| - |M_1|)\cdot(\partial V/\partial\Phi)} \qquad (3).$$

The value of $R_{d(SBC)}$ is determined by all the parameters, R_s, R_d, M_1, M_2, $\partial V/\partial\Phi$, and the choice of the working point.

[1] This is so also for most other preamplifiers.

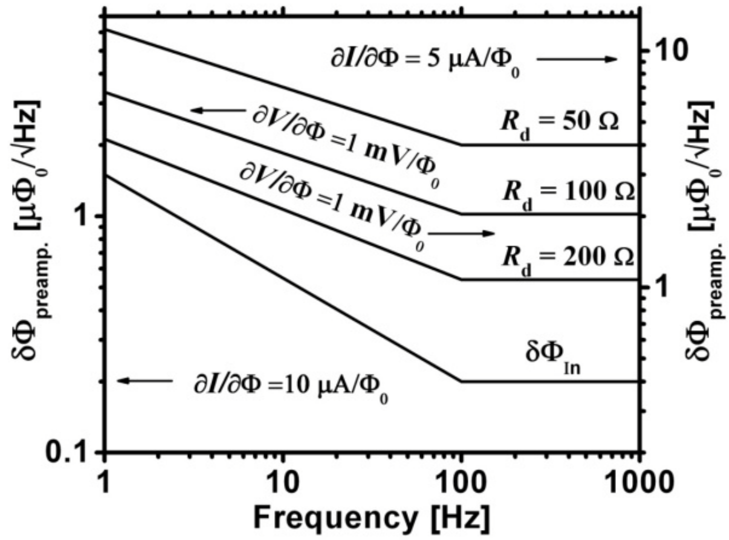

Fig. 2. Equivalent flux noise calculated for the preamplifier AD797 using its data sheet parameters. The lowest curve shows the net current noise contribution $\delta\Phi_{In}$.

III. EXPERIMENTAL RESULTS AND DISCUSSION

As in our already reported experiments [4,7] our SBC circuit was constructed of discrete components: a helium-cooled planar dc SQUID magnetometer with a loop inductance of 350 pH and a pickup area of 6 × 6 mm², a shunt resistor R_s, and two hand-wound coils L_1 and L_2 made of niobium wire. These two coils were independently coupled to the SQUID via the mutual inductances M_1 and M_2. The flux-to-field transfer coefficient $\partial B/\partial\Phi$ of this magnetometer was about 0.7 nT/Φ_0.To test the operation with the working point at W_1 and compare with that at W_2, we measured the I - Φ characteristic with M_1 = 0 and a moderate M_1 = ± 0.09 nH[2]. The obtained I - Φ plots are shown in Figure 3.

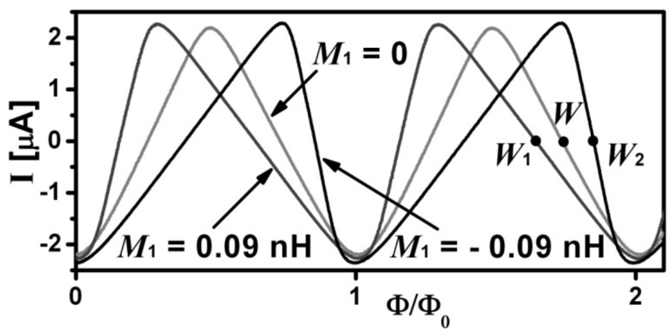

Fig. 3. The measured I - Φ characteristics for three different values of M_1. The three working points W_2, W, W_1 are shown in the middle of the negative slope.

The flux-to-current coefficients $\partial I/\partial\Phi_{(SBC)}$ obtained at working points W, W_1 and W_2 were: 11.8 μA/Φ_0 (W), 7.2 μA/Φ_0 (W_1) and 28 μA/Φ_0 (W_2).

To attain a sufficiently high $R_{d(SBC)}$ at W_1 [equation (3)] we chose a rather high M_2 of 3.2 nH and an appropriately high R_s of 250 Ω. The dynamic resistance determined from the slope

[2] Note that the sign of M_1 indicates the direction in which coils L_1 and L_2 are wound. Positive sign denotes the same direction, while negative sign stands for opposite directions.

of the I - V curve was 230 Ω at W_1, 120 Ω at W and 40 Ω at W_2. The criteria for choosing M_2 and R_s will be discussed elsewhere. Here it suffices to say that the contribution of Nyquist noise from the resistor R_s to the preamplifier noise was verified to be negligible in the range of 200-300 Ω. At all three working points the resulting $\partial V/\partial\Phi_{(SBC)} = R_{d(SBC)} \cdot \partial i/\partial\Phi_{(SBC)}$ reached 1 mV/Φ_0 so that the preamplifier noise was suppressed. Of course, at W our scheme effectively reduced to that of NC [3].

The measured SQUID noise spectra recorded in the three cases are plotted in Figure 4. The presumably intrinsic white SQUID noise level of our SQUID, $\sqrt{S_\Phi}$ = 2.5 μΦ_0/√Hz was achieved in all cases, corresponding to a field noise of our SQUID magnetometer equal $\sqrt{S_B}$ = 1.8 fT/√Hz. In the case of operation at W_1 we observed a slightly higher low-frequency noise. At present, we have no explanation of this slight increase, but speculate that it could be related to the large mutual inductance M_{12} between L_1 and L_2 when both coils are wound in the same direction.

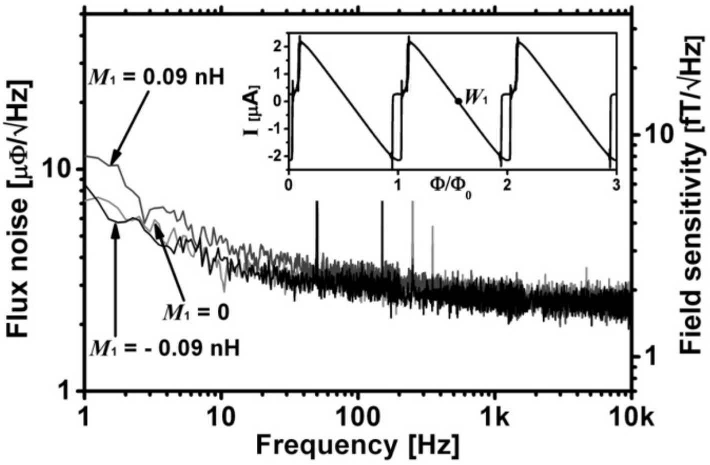

Fig. 4. Flux and field noise spectra measured with working points W, W_1 and W_2. The inset shows the hysteretic I - Φ curve for a larger M_1 = 0.17 nH resulting in $\partial I/\partial\Phi_{(SBC)} < 5$ μA/Φ_0, but no change in noise.

To test what happens when we increase the negative current feedback, we also increased M_1 to 0.17 nH, which reduced $\partial I/\partial\Phi_{(SBC)}$ to well below 5 μA/Φ_0. In this case hysteresis appeared on the very steep slope of the I - Φ curve and the current swing I_{pp} was somewhat reduced (see the inset in Figure 4). However, the measured noise spectrum remained the same as that measured with M_1 = 0.09 nH. This confirms experimentally the calculated level of the white preamplifier noise (Figure 2).

The main advantage of operating at W_1 should be the wide linearity range of the I - Φ characteristic, which in our experimentally tested case exceeds 0.5 Φ_0. This is illustrated by plots in Figure 5. While the upper plot is one of the curves of Figure 3, the lower is that of its derivative, *i.e.*, $\partial I/\partial\Phi_{(SBC)}$ versus Φ/Φ_0. So the exact setting of the working point is not critical and one can expect wide margins for all SBC components, *i.e.*, wide tolerance of parameters not affecting the performance of the SQUID scheme. Furthermore, a wide linear flux range improves not only the system linearity, but also the system slew rate $\dot{\Phi}_f$, because $\dot{\Phi}_f \propto \Phi_{lin}$ [8]. We are now investigating these issues and plan to present the results separately.

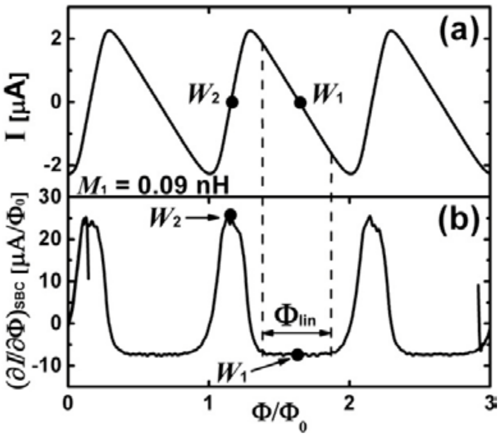

Fig. 5. Plots of I and $\partial I/\partial\Phi_{(SBC)}$ versus Φ/Φ_0 when M_1 = 0.09 nH. The width of the linear flux range Φ_{lin} (between the two dashed lines) where $\partial I/\partial\Phi_{(SBC)}$ is practically constant is 0.54 Φ_0.

IV. CONCLUSION

While the negative feedback has been reported and used before to enhance the linearity and slew rate of SQUID-array current sensor or two-stage SQUID array amplifiers [8, 9], we believe to be first in demonstrating that it can also be used to advantage in single-stage direct-coupled electronics potentially suitable for multichannel SQUID magnetometric systems. Provided that the required range of circuit parameters (a rather high M_2) can also be implemented on an integrated chip or in another technically convenient and inexpensive arrangement, the SBC might have bright future in multichannel SQUID systems as it should tremendously relax the low parameter spread requirement in the fabrication. Furthermore, one can infer from our preliminary data that also in the case of SQUIDs with much lower intrinsic noise $\sqrt{S_\Phi} < 1\ \mu\Phi_0/\sqrt{\text{Hz}}$, the scheme can provide low-noise performance making it suitable for future low-field MRI and other sensor array systems.

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