ESTIMATION OF UNCERTAINTY FOR FATIGUE GROWTH RATE AT CRYOGENIC TEMPERATURES

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ABSTRACT

Fatigue crack growth rate (FCGR) measurement data for high strength austenitic alloys at cryogenic environment suffer in general from a high degree of data scatter in particular at ΔK regime below 25 MPa√m. Using standard mathematical smoothing techniques forces ultimately a linear relationship at stage II regime (crack propagation rate versus ΔK) in a double log field called Paris law. However, the bandwidth of uncertainty relies somewhat arbitrary upon the researcher's interpretation. The present paper deals with the use of the uncertainty concept on FCGR data as given by GUM (Guidance of Uncertainty in Measurements), which since 1993 is a recommended procedure to avoid subjective estimation of error bands. Within this context, the lack of a true value addresses to evaluate the best estimate by a statistical method using the crack propagation law as a mathematical measurement model equation and identifying all input parameters. Each parameter necessary for the measurement technique was processed using the Gaussian distribution law by partial differentiation of the terms to estimate the sensitivity coefficients. The combined standard uncertainty determined for each term with its computed sensitivity coefficients finally resulted in measurement uncertainty of the FCGR test result. The described procedure of uncertainty has been applied within the framework of ITER on a recent FCGR measurement for high strength and high toughness Type 316LN material tested at 7 K using a standard ASTM proportional compact tension specimen. The determined values of Paris law constants such as C_0 and the exponent m as best estimate along with the their uncertainty value may serve a realistic basis for the life expectancy of cyclic loaded members.

KEYWORDS: Uncertainty, 316LN, fatigue crack growth rate, cryogenics
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INTRODUCTION

Regarding the nature of measurements, so far every measurement bears some degree of uncertainty. The FCGR values belong also to a sort of data, which exhibit in majority of cases, in particular at cryogenics, much scatter. However, beside the instrumental uncertainties a great deal of the measurement uncertainties results in itself from the material. Small inclusions and inclusion spacing may affect much the FCGR behaviour [1]. In general, at low ΔK levels (20 - 30 MPa√m) crack propagation can be retarded or accelerated at different crack growth lengths. Even in some cases according to the material condition it may also retard or accelerate at high ΔK levels. Part of the retardation supposed to be attributed for the so called crack closure effects [2, 3]. Therefore, the acquired raw data, which is in that case the crack growth versus cycle number, reflects the situation correctly as a natural scatter, apart the measurement errors. The FCGR data usually are represented in form of crack growth rate versus stress intensity range ΔK in a double log graph. To illustrate the problems Figure 1 (a) shows the record of a fatigue crack growth test for a Type 316LN plate material in rolled condition. As obvious the differentiation of these raw data in form of da/dN will result in partly a negative crack growth with respect to the recorded data. To avoid this, some sort of mathematical handling is necessary, in particular for the double log graph da/dN versus ΔK as here negative values can not be handled. The reference gives some valuable information about the handling of data scatter and about the smoothing techniques [4, 5]. In Figure 1 (b) the evaluated plots after smoothing are given along with the numerical differentiated raw data, by excluding the negative data.

Beside these difficulties welded material pairs depending on the crack growth path inside the weld zone may show peculiar phenomenon during the FCGR test. Such a test result with a Nitronic 50 material determined at 7 K is given in Figure 2, where the crack propagation was inside the welded double U-joint specimen, which was machined out of 60 mm thick plate. The crack, starting from the root section penetrated entirely inside the weld metal and tracked to the weld surface. The unusual record in contrast to Figure 1 (a) confirms initially a large influence of base/weld material portion along with probable existing residual stresses. The record of crack length versus cycle number reveals after the differentiation a negative slope up to ΔK ~30 MPa√m inside the double log graph (see Figure 2 (b)). Later as the crack penetration is fully inside the weld zone the crack grows in a standard monotonic way similar to the crack propagation in a symmetrical weld zone such as e.g. in a narrow weld seam in weld longitudinal orientation. Again in Figure 2 (b) the raw data and the smoothed plots are given and the arrows indicate the two different regions.

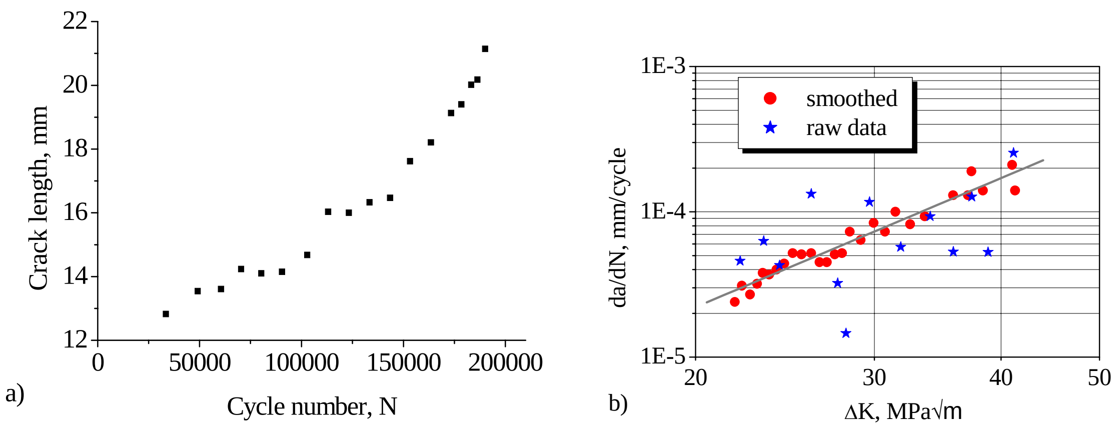

FIGURE 1 (a) Record shows the crack length versus cycle number of a 316LN rolled plate in transverse orientation measured at 7 K. (b) The graph shows the fatigue crack growth rate diagram for raw data as well as the smoothed record.

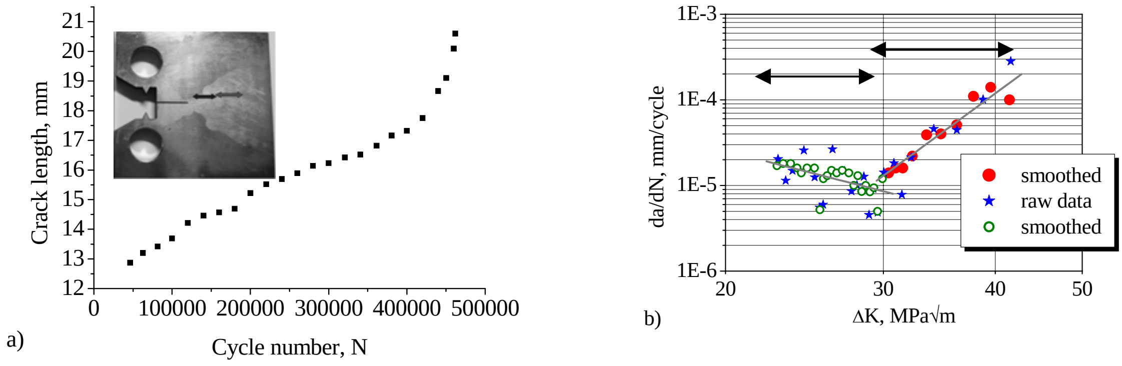

FIGURE 2 a) Record shows the crack length versus cycle number of a 60 mm thick welded section with the material Nitronic 50 measured in weld longitudinal orientation at 7 K. b) This graph shows the fatigue crack growth rate diagram for the raw data as well as the smoothed record. FCGR starts from root of the weld (arrow at left), decreases initially and increases later during further crack propagation inside the upper weld region (arrow at right).

According to this finding it can be assumed that the root region has more or less the performance of the base metal, whilst the weld metal has usually a significant low FCGR. Regarding these entire scatter of FCGR data a factor of 1.5 on da/dN values can be seen as an acceptable level of the uncertainty although a rigorous uncertainty calculation is still lacking. Only for room temperature tests a reference [6] exist for uncertainty estimation.

Back in 1995, a number of international standards organizations, decided to unify the use of statistical terms in their standards. It was decided to use the word “uncertainty” for all quantitative (associated with a number) statistical expressions and eliminate the quantitative use of “precision” and “accuracy.” The words “accuracy” and “precision” are allowed to be still used qualitatively. The terminology and methods of uncertainty evaluation are standardized in the Guide to the Expression of Uncertainty in Measurement (GUM) [7]. The essence of this concept is to determine the best estimate of the parameter, here the rate da/dN, as there is no possibility to obtain the true value. References [8-12] give further details about this concept. Even in an inter laboratory round robin test the true value can not be determined as each laboratory uses his best technique according to their set up and knowledge status. Therefore, the uncertainty of the best estimate is than a function of the combined standard uncertainty associated with the model equation, which in case of FCGR is given by the following Paris law equation:

$$\frac{\partial a}{\partial N} = C_0 \cdot \Delta K^m \tag{1}$$

The stress intensity range ΔK is given according to the ASTM 1820 as follows:

$$\Delta K = \frac{\Delta P}{B \cdot \sqrt{W}} \cdot \frac{\left(2 + \frac{a}{W}\right)}{\left(1 - \frac{a}{W}\right)^{1.5}} \cdot \left(0{,}886 + 4{,}64 \cdot \frac{a}{W} - 13{,}32 \cdot \left(\frac{a}{W}\right)^2 + 14{,}72 \cdot \left(\frac{a}{W}\right)^3 - 5{,}6 \cdot \left(\frac{a}{W}\right)^4\right) \tag{2}$$

Taking the equations (1) and (2) the FCGR is simply a function of following variables, where R is the rate of the crack growth under cyclic loading:

$$\frac{\partial a}{\partial N} = R = f\left(C_0, m, \Delta P, W, B, a\right) \tag{3}$$

Here C_0 and m are Paris equation constants, ΔP the cyclic load range, W the width, B the thickness, and "a" the crack length of the compact tension specimen, respectively. By partial differentiation of these six parameters and sorting the uncertainty terms of each one it is thus possible to define a reliable combined uncertainty value according to the concept as given by GUM [7].

DETERMINATION PROCEDURE OF UNCERTAINTY FOR FCGR

The cyclic crack growth law given in equation (1) is the model equation with the variables as given in equation (3). This model equation describes the actual law of the FCGR and for the determination of the uncertainty this equation had to be processed by partial differentiation of each variable. The combination of all terms according to the Gaussian distribution law results in a combined standard uncertainty. The following equation shows the combined standard uncertainty u_c for the rate R, where the terms $u_1 \cdots u_6$ the uncertainties of each variable are, which will be defined later.

$$u_c = \sqrt{\left(\frac{\partial R}{\partial C_0}\right)^2 u_1^2 + \left(\frac{\partial R}{\partial m}\right)^2 u_2^2 + \left(\frac{\partial R}{\partial \Delta P}\right)^2 u_3^2 + \left(\frac{\partial R}{\partial W}\right)^2 u_4^2 + \left(\frac{\partial R}{\partial B}\right)^2 u_5^2 + \left(\frac{\partial R}{\partial a}\right)^2 u_6^2} \quad (4)$$

The partial differential of these variables called as sensitivity coefficients are given:

$$\frac{\delta R}{\delta C_0} = \Delta K^m \quad and \quad \frac{\delta R}{\delta m} = C_0 \cdot \Delta K^m \cdot \ln \Delta K \quad (5)$$

$$\frac{\partial R}{\partial \Delta P} = C_0 \cdot \Delta K^m \cdot \frac{m}{\Delta P} \quad and \quad \frac{\partial R}{\partial B} = -C_0 \cdot \Delta K^m \cdot \frac{m}{B} \quad (6)$$

$$\frac{\partial R}{\partial W} = C_0 \cdot \Delta K^m \cdot m \cdot \frac{\left[F_A + F_B + F_C + F_D\right]}{\Delta P} \cdot B \cdot F_E \quad (7)$$

$$\frac{\partial R}{\partial a} = C_0 \cdot \Delta K^m \cdot m \cdot \frac{\left[-\frac{2 \cdot F_A}{\left(2+\frac{a}{W}\right)} + F_F - \frac{F_D \cdot \sqrt[5]{W}}{\sqrt[3]{W}}\right]}{\Delta P} \cdot B \cdot F_E \quad (8)$$

Where the functions F_A, F_B, F_C, F_D, F_E, and F_F are given below:

$$F_A = -\Delta P \cdot \left(2 + \frac{a}{W}\right) \cdot \frac{\left(8.886 + 4.64 \cdot \left(\frac{a}{W}\right) - 13.32 \cdot \left(\frac{a}{W}\right)^2 + 14.72 \cdot \left(\frac{a}{W}\right)^3 - 5.6 \cdot \left(\frac{a}{W}\right)^4\right)}{\left(1 - \frac{a}{W}\right)^{1.5} \cdot 2 \cdot B \cdot \sqrt[3]{W}} \quad (9)$$

$$F_B = -\Delta P \cdot a \cdot \frac{\left(8.886 + 4.64 \cdot \left(\frac{a}{W}\right) - 13.32 \cdot \left(\frac{a}{W}\right)^2 + 14.72 \cdot \left(\frac{a}{W}\right)^3 - 5.6 \cdot \left(\frac{a}{W}\right)^4\right)}{\left(1 - \frac{a}{W}\right)^{1.5} \cdot B \cdot \sqrt[5]{W}} \quad (10)$$

$$F_C = \Delta P \cdot \frac{\left(2 + \frac{a}{W}\right) \cdot \left(-4.64 \cdot \frac{a}{W^2} + 26.64 \cdot \frac{a^2}{W^3} - 44.16 \cdot \frac{a^3}{W^4} + 22.4 \cdot \frac{a^4}{W^5}\right)}{\left(1 - \frac{a}{W}\right)^{1.5} \cdot B \cdot \sqrt{W}} \quad (11)$$

$$F_D = -1.5 \cdot \Delta P \cdot \frac{\left(2 + \frac{a}{W}\right) \cdot \left(8.886 + 4.64 \cdot \left(\frac{a}{W}\right) - 13.32 \cdot \left(\frac{a}{W}\right)^2 + 14.72 \cdot \left(\frac{a}{W}\right)^3 - 5.6 \cdot \left(\frac{a}{W}\right)^4\right)}{\left(1 - \frac{a}{W}\right)^{2.5} \cdot B \cdot \sqrt[5]{W}} \quad (12)$$

$$F_E = \frac{\sqrt{W}}{\left(8.886 + 4.64 \cdot \frac{a}{W} - 13.32 \cdot \left(\frac{a}{W}\right)^2 + 14.72 \cdot \left(\frac{a}{W}\right)^3 - 5.6 \cdot \left(\frac{a}{W}\right)^4\right)} \cdot \frac{\left(1 - \frac{a}{W}\right)^{1.5}}{\left(2 + \frac{a}{W}\right)} \quad (13)$$

$$F_F = \Delta P \cdot \frac{\left(2 + \frac{a}{W}\right) \cdot \left(4.64 \cdot \frac{1}{W} - 26.64 \cdot \frac{a}{W^2} + 44.16 \cdot \frac{a^2}{W^3} - 22.4 \cdot \frac{a^3}{W^4}\right)}{\left(1 - \frac{a}{W}\right)^{1.5} \cdot B \cdot \sqrt{W}} \quad (14)$$

In following the application of the uncertainty concept is given for the rolled plate material, which is shown in Figure 1 (b). The linear regression analysis in the double log graph result in an usual statistical 1st order polynomial fit estimate with Paris law coefficients of C_0= 3.226.10^{-9} and m = 2.949 with a square root of regression coefficient R^2 of 94.4 %. These Paris coefficients along with the geometrical values of the compact tension specimen are necessary to compute a distinct value on the FCGR line. The specimen geometrical conditions and the applied cyclic load range are as follows:

a = 1.5 cm, W = 3.6 cm, B = 0.4 cm, and ΔP = 2.5 kN
(15)

Here the crack length has been arbitrary selected as a value of 15 mm to be able to compute a numerical value for stress intensity range ΔK and the da/dN on the fitted line. Inserting all these input data given in (15) into the equation (2) the computed value of ΔK is 25 MPa√m. Inserting now all specimen geometrical data and using the equations (5 to 14) results in partial differential terms, which is given in Table 1 for the anticipated crack length of 15 mm. Table 1 shows the compilation of computed values in form of partial differentiations.

TABLE 1 Computed values of partial differentials for the FCGR test of the line given in Figure 1 (b)

$\frac{da}{dN}$	ΔK	$\frac{\partial R}{\partial C_0}$	$\frac{\partial R}{\partial m}$	$\frac{\partial R}{\partial \Delta P}$	$\frac{\partial R}{\partial W}$	$\frac{\partial R}{\partial B}$	$\frac{\partial R}{\partial a}$
$4.311 \cdot 10^{-5}$	25.066	$1.336 \cdot 10^{4}$	$1.918 \cdot 10^{-4}$	$5.085 \cdot 10^{-5}$	$-5.724 \cdot 10^{-5}$	$-3.178 \cdot 10^{-4}$	$9.50 \cdot 10^{-5}$

The uncertainty u_x of each variable referring to equation (3) are the next step of the uncertainty calculation. For the measurements given in Figure 1 the used load cell has the following specification as shown in Table 2.

TABLE 2 Force transducer specifications according to manufacturer's (MTS: 661.20) data sheet

Load cell capacity, N	Hysteresis % / full scale	Temperature coeff. on zero % / K	Temperature coeff. on sensitivity % / K
25000	0.05	0.002	0.002

According to this specification, the data should be converted to standard uncertainty values before combining them. These data are treated as Type B uncertainties because these are not been obtained from repeated observations. The temperature range between 295 K and 288 K has been selected to reflect the conditions of the cryogenic test facility during the possible environmental temperature variation. The following equation for the load describes the situation for the possible force transducer uncertainty sources, which includes the three terms of error taken from Table 2.

$$P = hysteresis + T_{CoeffonZero} + T_{CoeffonSens} \tag{16}$$

The percentage specifications are converted to load units based on corresponding input value of ΔP = 2500 N necessary for the selected ΔK from Figure 1 (b) and the input data of (15). Thereafter, the values are converted to standard uncertainties assuming a rectangular distribution Type B where the combined standard uncertainty u_p for the load cell is:

$$u_P = \sqrt{\left(\frac{hysteresis \cdot 2500}{100 \cdot \sqrt{3}}\right)^2 + \left(\frac{T_{CoeffonZero} \cdot 2500}{100 \cdot \sqrt{3}}\right)^2 + \left(\frac{T_{CoeffonSens} \cdot 2500}{100 \cdot \sqrt{3}}\right)^2} = 0.776 \quad N \tag{17}$$

Therefore, the uncertainty for ΔP is given according to Type B concept as follows:

$$u_3 = 0.776 \quad N \quad or \quad \approx \quad 0.0008 \quad kN \tag{18}$$

In contrast to ΔP the quantities W and B are determined using repeated observations and therefore these estimates can be settled as a Type A whose value can be given as standard deviation divided by square root of tests, which is obtained by repeated measurements. The determined uncertainties for W and B here are u_4 = 0.002 mm and u_5 = 0.002 mm, respectively. For the crack length the estimation according to the recent measurements show a maximum of 0.4 mm deviation between the observed value of compact tension specimen after fracturing it into two halves measured by microscope and the measured crack lengths during the test using the compliance technique with an extensometer. In this case the rectangular distribution Type B has been applied similar to equation (18), which results in a uncertainty term of u_6 = 0.115 mm.

Differently to the geometrical uncertainties, which were all handled as Type B or Type A distribution the experimental uncertainties of the tests are embedded in the final results. The test given in Figure 1 had been performed with two identical specimens. Considering this the gathered FCGR data of these tests recently conducted within the framework of ITER is given in Table 3. The compiled results of C_0 and m for each data set of two specimens from an identical batch has been evaluated considering their variation and

TABLE 3 Computed values of C_0 and m for two identical specimens in longitudinal and transverse orientations of the same batch after the carried out FCGR test at 7 K

Specimen, condition, & code	C_0, specimen # 1	C_0, specimen # 2	absolute difference	m, specimen # 1	m, specimen # 2	absolute difference
316LN rolled plate, longitudinal	$1.79\cdot10^{-9}$	$2.65\cdot10^{-9}$	$8.64\cdot10^{-10}$	3.092	3.012	0.080
316LN rolled plate, transversal	$3.23\cdot10^{-9}$	$1.37\cdot10^{-9}$	$1.86\cdot10^{-9}$	2.949	3.193	0.244

by assuming that the experimental error is identical for all four tests as the uncertainties resulting from the geometrical constraints has been considered elsewhere.

Following this the averages of differences of these four measurements at 7 K performed with the same batch of the material with respect to C_0 and m are $1.362\cdot10^{-9}$ and 0.162, respectively. Using the concept of Type B (mean/2/√3) distribution it has been determined that the experimental values according to GUM results in uncertainty terms for C_0 and m as $3.93\cdot10^{-10}$ and 0.047, respectively. The combined standard uncertainty of this test using the equation (3) and all computed values gives thus as follows:

$$u_c = \sqrt{\left(1.336\cdot10^{4}\right)^2\cdot\left(3.93\cdot10^{-10}\right)^2+\left(1.918\cdot10^{-4}\right)^2\cdot(0.047)^2+\left(5.085\cdot10^{-5}\right)^2\cdot(0.0008)^2+\left(-5.724\cdot10^{-5}\right)^2\cdot(0.002)^2+\left(-3.178\cdot10^{-4}\right)^2\cdot(0.002)^2+\left(9.5\cdot10^{-5}\right)^2\cdot(0.115)^2}\quad \frac{mm}{cycle} \qquad (20)$$

$$u_C = 1.51\cdot10^{-5} \quad \frac{mm}{cycle} \qquad (21)$$

Finally this results in:

$$\frac{da}{dN} = R = 4.311\cdot10^{-5} \quad \pm \quad 1.51\cdot10^{-5} \quad \frac{mm}{Cycles} \quad for \quad \Delta K = 25.07 \quad MPa\sqrt{m} \qquad (22)$$

DISCUSSION

The determined result of uncertainty for the rolled stainless steel given in Figure 1 (b) shows an uncertainty bandwidth of the line, which was obtained with the usual regression analysis a value of $\pm1.51\cdot10^{-5}$ mm/cycles referred to the position of ΔK = 25 MPa√m. A parallel shift of the best estimated line (±) with respect to this ΔK value will result in the region of FCGR uncertainty. Knowing this, one may conclude that in a sound laboratory environment even with proper instrumentation this error bandwidth can not be reduced much. Therefore, for the design of heavy cyclic loaded critical components at 4 K the life expectancy computation should take as input data the shifted upper parallel line of the FCGR data to cover the safety requirements.

CONCLUSIONS

This work shows the necessary steps for the calculation of the uncertainty following the cryogenic fatigue crack growth rate measurement result of a rolled Type 316LN stainless steel. The calculation of the uncertainty value has been carried out using the statistical concept as given by GUM. The model equation which is the equation of the Paris line inside the double log graph has been differentiated for each variable to obtain the necessary sensitivity coefficients. For the two Paris constants C_0 and exponent m four independent FCGR tests with the specimens of the same batch have been taken to obtain the experimental variation. The combined standard uncertainty finally resulted in an estimation of $\pm 1.51 \cdot 10^{-5}$ mm/cycles. This value is suggested to be the best estimate and can be used as a bandwidth for the obtained FCGR line to cover the necessary safety for the material under design.

REFERENCES

1. Read D. T. and Reed R. P., " Toughness, fatigue crack growth, and tensile properties of three nitrogen-strengthened stainless steels at cryogenic temperatures*", in Technical Reports*, NBSIR 78-884 Material studies for magnetic fusion energy applications at low temperatures, 1978, pp. 93-154
2. Jono M. and Song J., " Fatigue crack growth characteristics and crack closure of structural materials", *in Fatigue 84*, 2nd Int. Conf. on Fatigue and fatigue thresholds, September 1984, pp. 717-726
3. Tokaji K., Ando Z., Nagae K., and Imai T. "Effect of sheet thickness on fatigue crack retardation and validity of crack closure concept", *in Fatigue 84*, 2nd Int. Conf. on Fatigue and fatigue thresholds, September 1984, pp. 727-737
4. Munro H. G., , "The determination of fatigue crack growth rates by a data smoothing technique", Royal Military College of Science, 1973, *in Int. Journal of Fracture* 9
5. Zheng J. and Powell B. E.**, "A** method to reduce the scatter in fatigue crack growth rate data", *in Fatigue Fract.* **Engng *Mater.*** *Struct.* Vol. 20, No. 9, 1997, **pp.** 1341-1350
6. Georgsson P., "The Determination of Uncertainties in Fatigue Crack Growth Measurement", Volvo Aero Corporation, Trollhättan, in Standards Measurement & Testing Project No. SMT4-CT97-2165, UNCERT COP 05: 2000
7. BIPM, IEC, IFCC, ISO, IUPAC and OIML, "Guide to the expression of uncertainty in measurement", 1995
8. TAYLOR, B.N. and KUYATT, C.E. "Guidelines for Evaluating and Expressing the Uncertainty of NIST Measurement Results", NIST Technical Note 1297, 1994
9. Kragten J., "Calculating standard deviations and confidence intervals with a universally applicable spreadsheet technique", *Analyst*, **119**, 2161-2166 (1994)
10. EURACHEM / CITAC Guide CG 4, "Quantifying Uncertainty in Analytical Measurement", Editors S L R Ellison (LGC, UK), M Rosslein (EMPA, Switzerland), A Williams (UK), Second edition 2000
11. Churchill E., Harry H. Ku., and Colle R.**,** "Expression of the Uncertainties of Final Measurement Results: Reprints", NBS Special Publication 644, National Bureau of Standards, Washington, DC (1983)
12. The Japan Accreditation Board for Conformity Assessment, "Estimation of Measurement Uncertainty (Electrical Testing / High Power Testing)," JAB NOTE4 Edition 1, March 2003