IEEE/CSC & ESAS SUPERCONDUCTIVITY NEWS FORUM (global edition), April 2015.
Tutorial presentation given at KRYO 2014, Berlin, September 21-23, 2014.

Physikalisch-Technische Bundesanstalt
Braunschweig und Berlin
Nationales Metrologieinstitut

Thermometry at low temperature

Alexander Kirste

Kryo 2014, Berlin, 21 September 2014

Outline

- Thermometer types, properties
- Thermal contact

- PLTS-2000, dissemination of the kelvin
- ^{3}He melting curve thermometer (MCT)
- Coulomb blockade thermometer (CBT)
- Nuclear orientation thermometer

- Noise thermometers
  - Josephson junction noise thermometer (JNT)
  - Current sensing noise thermometer (CSNT)
  - Magnetic field fluctuation thermometer (MFFT)

- Conclusion

Thermometer Types

Thermometer function: $m = f(T, x_1, x_2, ...)$ T - temperature
x_i - parameter i

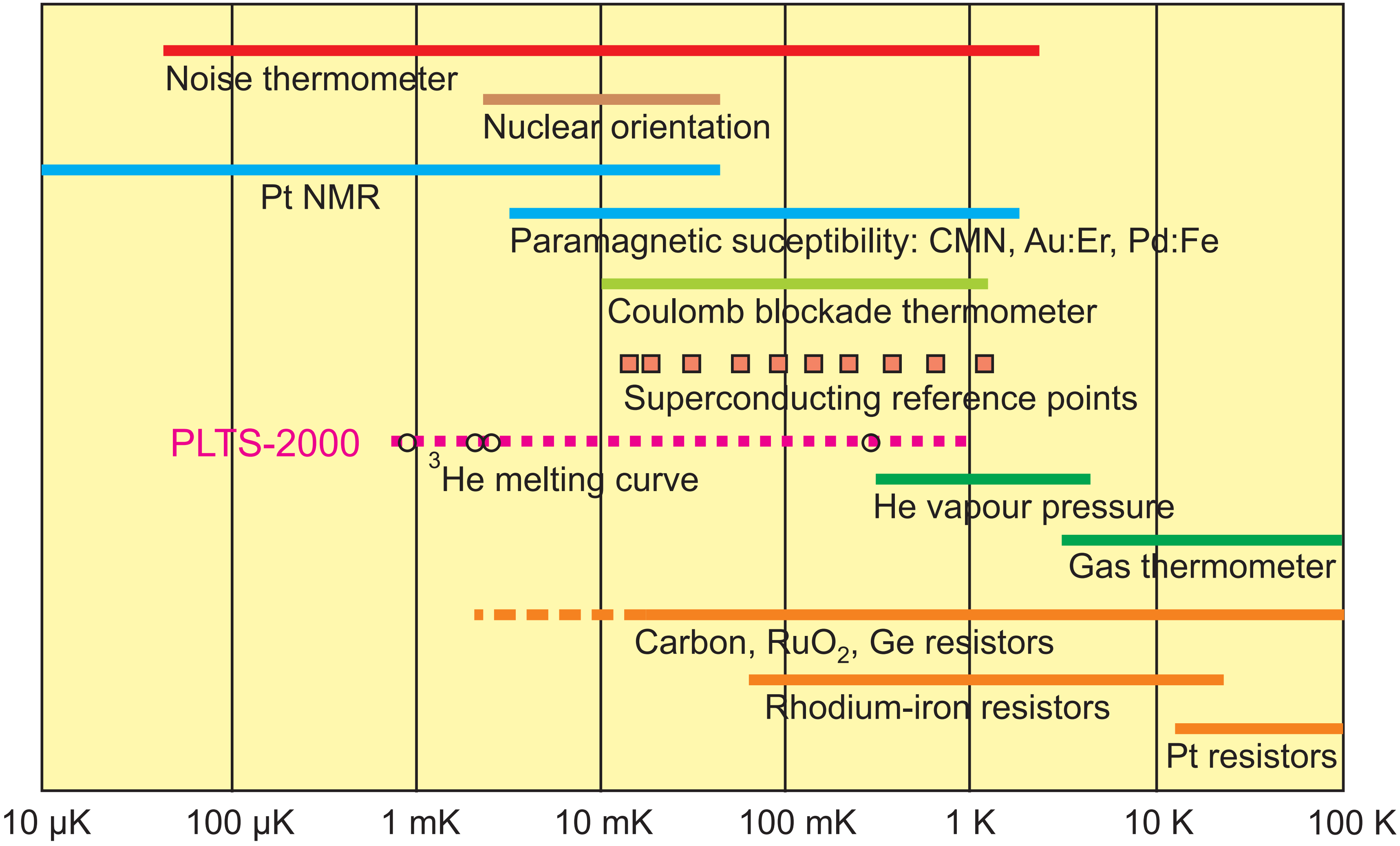
Primary thermometer:
- functional dependence $f(...)$ is known
- all other parameters x_i are known (might be determined without knowing T)

⇒ can be used without calibration

Secondary thermometer:
- first kind: functional dependence $f(...)$ is known, but one or more x_i are unknown
- second kind: no physically founded functional dependence $f(...)$ is known

⇒ must be calibrated at known T

Low Temperature Thermometers



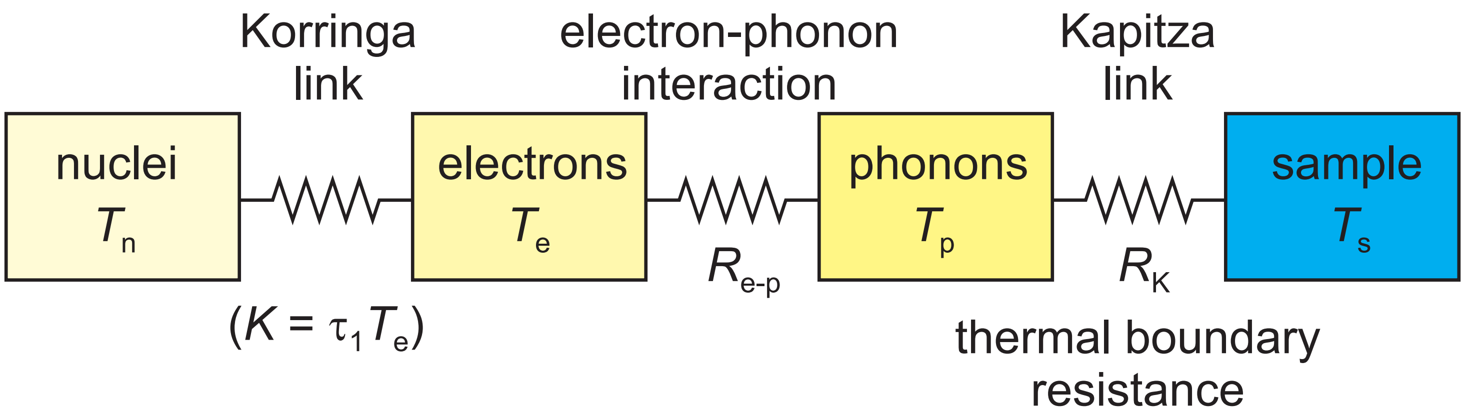
Properties of Thermometers

- sensitivity: $(\Delta f/f) / (\Delta T/T)$
- experimentally easy measurement of $f(T, x_1, x_2, ...)$
- relaxation times to reach thermal equilibrium
  - internal: spin-lattice relaxation time τ_1, spin-spin relaxation time τ_2
  - external: heat capacity, thermal conductivity, thermal contact
- power necessary to read out the device $\Rightarrow$ dissipation & self heating

- speed of the thermometer: time to reach a given uncertainty $u(T)$

- long-term stability: drift of parameters x_i
- stability with respect to thermal cycling
- external conditions: e.g. magnetic field B can affect $f(...)$ (or B is required),
  vibrations,
  rf interference

Thermal Contact



Korringa link

electron-phonon interaction

Kapitza link

nuclei T_n

electrons T_e

phonons T_p

sample T_s

$(K = \tau_1 T_e)$

$R_{e\text{-}p}$

R_K

thermal boundary resistance

Heat flow $\dot{Q}$ through thermal resistance R causes **temperature step** ΔT:

$$\dot{Q} = \Delta T / R$$

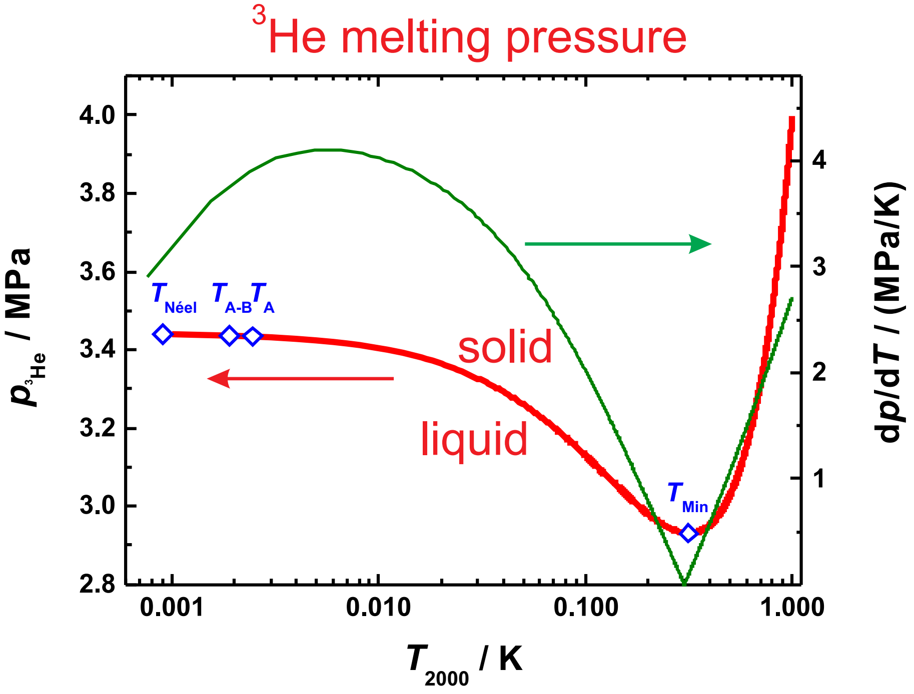
Metallic contact: **Wiedemann-Franz law**, relating thermal & electrical conductivities
κ σ

$$L_0 = \kappa / (\sigma T) = 2.45 \cdot 10^{-8}\ \mathrm{W\Omega K^{-2}}$$

e.g. 0.1 μΩ at 4 K for bolted contacts

The International Temperature Scale PLTS-2000

- PLTS-2000: **P**rovisional **L**ow **T**emperature **S**cale of 2000
- adopted by the International Committee for Weights and Measures (CIPM - *Comité International des Poids et Mesures*)



$$\frac{p_{3\mathrm{He}}}{\mathrm{MPa}} = \sum_{i=-3}^{9} a_i \left(\frac{T_{2000}}{\mathrm{K}}\right)^i$$

***T* range: 0.9 mK to 1 K**

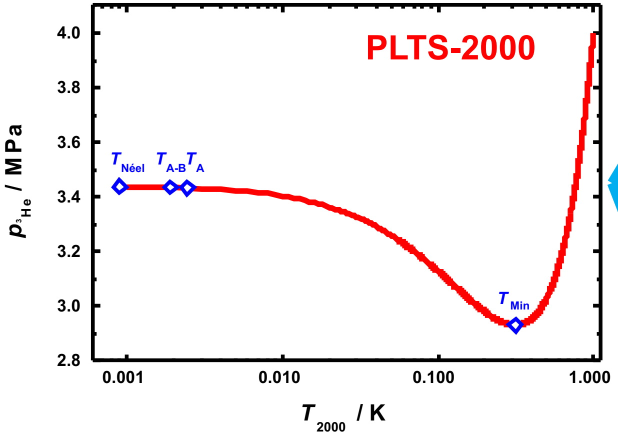
p range: 2.9 MPa to 4 MPa

Fixed points	p_{3He} / MPa	T_{2000} / mK
Minimum	2.93113	315.24
A	3.43407	2.444
A-B	3.43609	1.896
Neél	3.43934	0.902

- 4 inherent fixed points of the scale:
  3 phase transitions (A, A-B, Neél) besides the minimum of the melting curve $p_{3He}(T_{2000})$

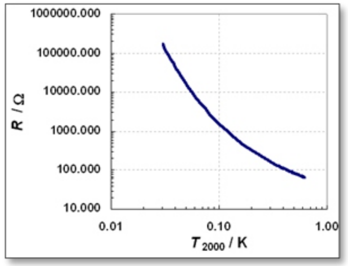
Dissemination of the Kelvin

Legal task of PTB:
realisation and dissemination of SI units (m, kg, s, A, K, mol, cd)

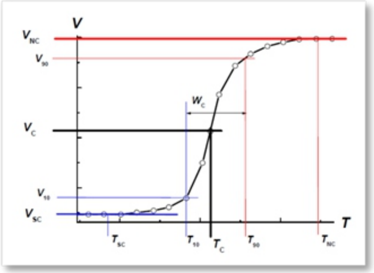


⇒ **Dissemination** of the international temperature scale **PLTS-2000** by calibration of practical thermometers for users worldwide

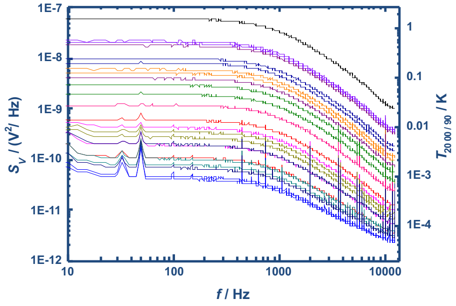
Resistance thermometers



Superconducting reference point samples



Noise thermometers: MFFT, CSNT



Physikalisch-Technische Bundesanstalt
Braunschweig und Berlin

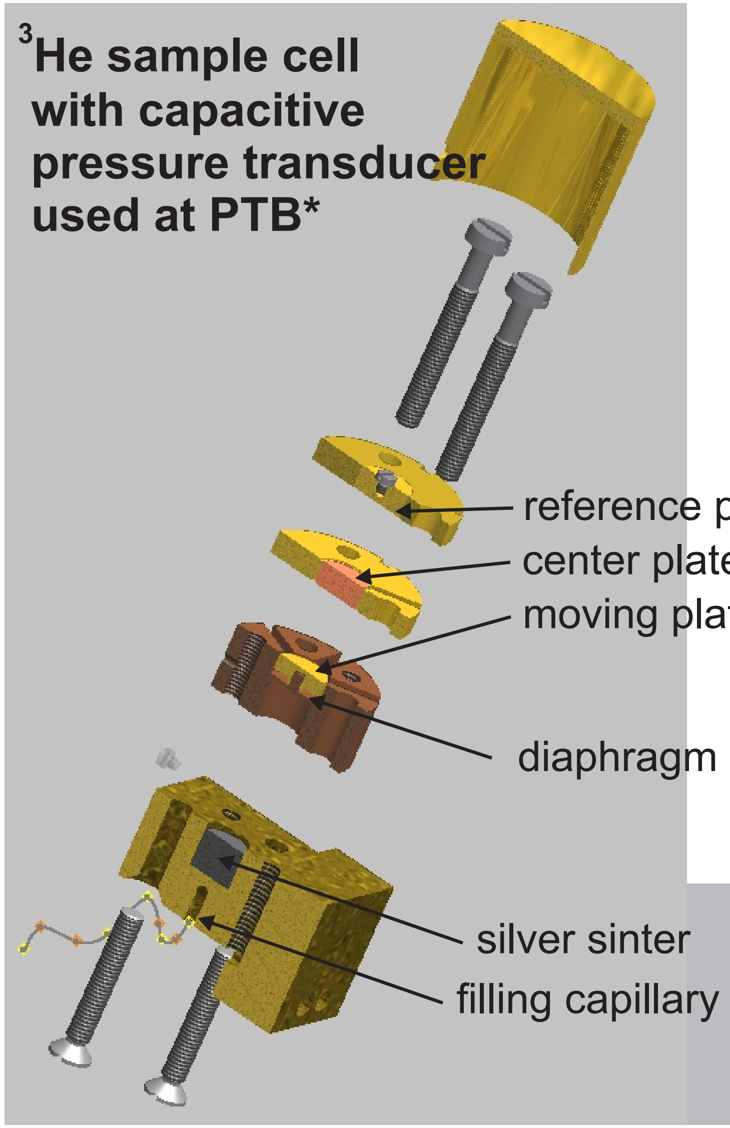
Kalibrierschein
Calibration Certificate

Gegenstand: Magnetic-Field-Fluctuation Thermometer (MFFT)
Hersteller: Magnicon GmbH
Typ: MFFT-1 Noise Thermometer
Kennnummer: [Nummer des untersuchten Gerätes, Normals, Präparats]
Auftraggeber: Magnicon GmbH, Lemsahler Landstr. 171, 22397 Hamburg, Germany, www.magnicon.com
Anzahl der Seiten: 5
Geschäftszeichen: [YYY-ZZZZZZZZ]
Kalibrierzeichen: [YYY-XXXXXXX]
Datum der Kalibrierung: [Datum oder Beginn bis Ende der Kalibrierung]
Im Auftrag Berlin, Im Auftrag
Siegel

calibration certificate

Implementation of the ^{3}He MCT (PLTS-2000)

^{3}He sample cell with capacitive pressure transducer used at PTB*



- laboratory standard in specialist labs
- field independent

- considered as difficult to implement for expanding community of users of ultra-low temperature platforms

- requires calibration of the pressure transducer
- measurement of absolute pressure
- slow traversing of fixed points

⇒ expert knowledge and experience required

*For details see:
G. Schuster, A. Hoffmann and D. Hechtfischer, Realisation of the temperature scale PLTS-2000 at PTB, PTB, Braunschweig, PTB-ThEx-21, 29pp, 2001, available through www.ptb.de

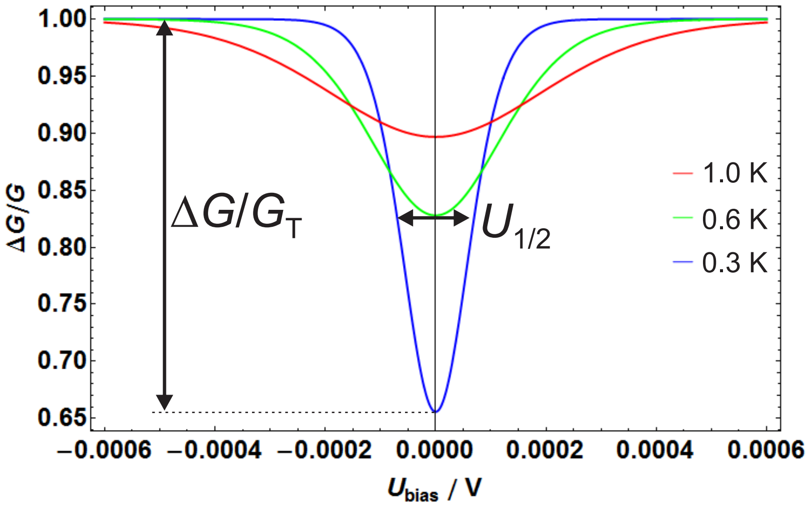
Coulomb Blockade Thermometry

Aalto University
School of Science *Low Temperature Laboratory, PICO-group*

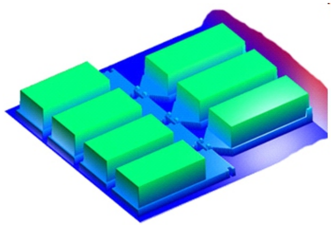
- arrays of tunnel junctions
- weak Coulomb blockade: $E_C << k_B T$
- measure differential conductance $\Delta G/G_T$
- primary or secondary thermometer mode

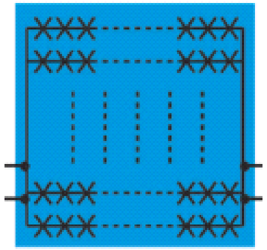
$$\frac{\Delta G}{G_T} = 1 - \frac{2(N-1)}{N}\frac{E_C}{k_B T}\left(\frac{x\sinh(x) - 4\sinh^2(x/2)}{8\sinh^4(x/2)}\right)$$

$$T = \frac{1}{5.439}\frac{eU_{1/2}}{Nk_B} \qquad x = \frac{eU_{\text{bias}}}{Nk_B T}$$



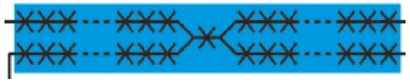
Coulomb blockade thermometer (CBT)

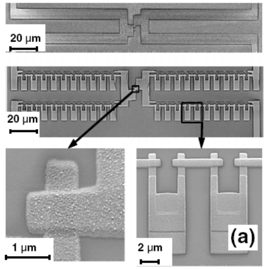




J. Pekola et al., Phys. Rev. Lett. 1994,
„Thermometry by Arrays of Tunnel Junctions“

Single junction thermometer (SJT)

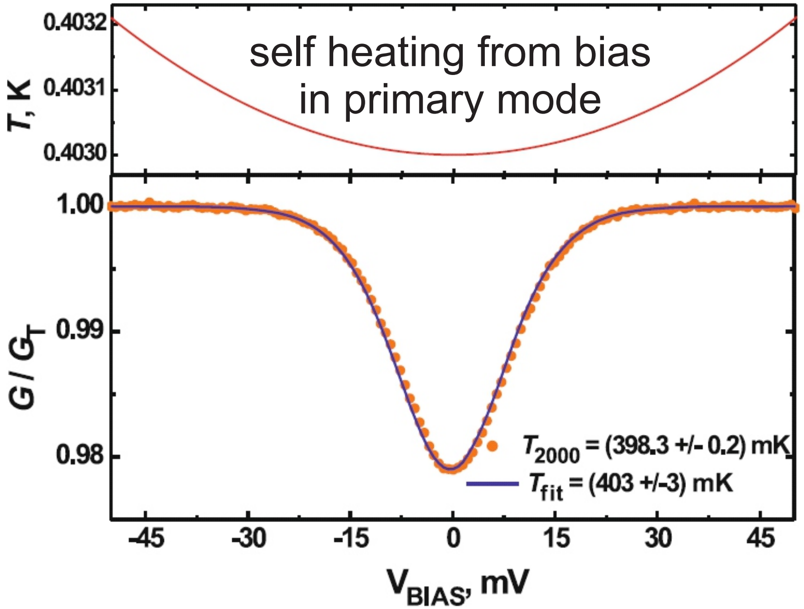




J. Pekola et al., Phys. Rev. Lett. 2008,
„Primary tunnel junction thermometry“

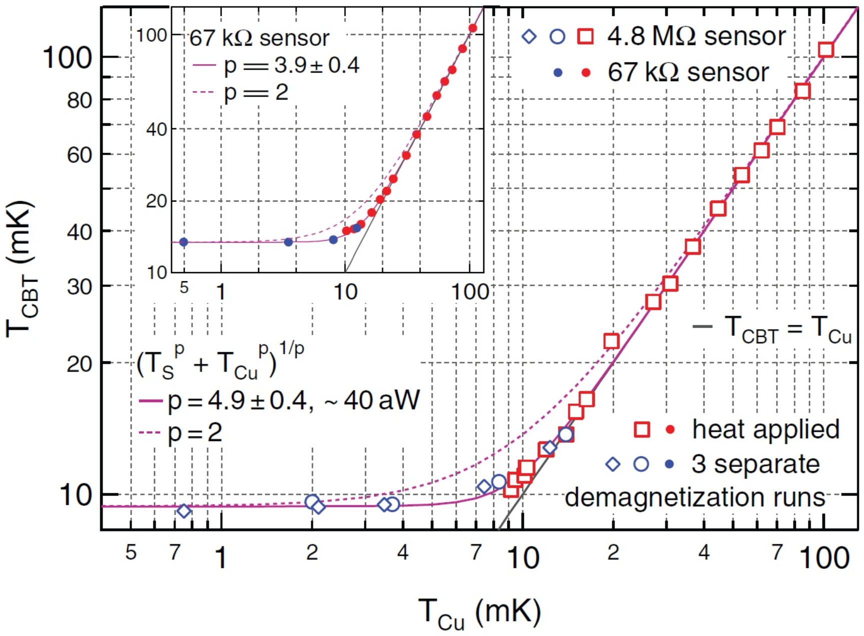
Coulomb Blockade Thermometry

(a) Experimental data at ≈400 mK



Meschke et al, Int. J. Thermophys. 2011

Thermal decoupling



Casparis et al, Rev. Sci. Instrum. 83 (2012) 083903

- no influence of magnetic field on T_{CBT}
- limitations:
  - inhomogeneities in the array (varying junction resistance)
  - high T: signal too small to measure if $\Delta G/G_T < 0.003$ (~ 30 K)
  - low T: increasing sensitivity to background charges if $\Delta G/G_T > 0.3$ (SET), hot-electron effect

Coulomb Blockade Thermometry

- Full references:
  J.P. Pekola, K.P. Hirvi, J.P. Kauppinen, and M. A. Paalanen, Thermometry by Arrays of Tunnel Junctions, Phys. Rev. Lett. **73** (1994) 2903
  J.P. Pekola, T. Holmqvist, and M. Meschke, Primary Tunnel Junction Thermometry, Phys. Rev. Lett. **101** (2008) 206801

- Meaning of symbols:
  N - number of tunnel junctions in series
  G_T - asymptotic conductivity at high bias voltages U_{Bias}
  $E_C = e^2 / 2C_{\mathrm{eff}}$ - charging energy, with C_{eff} being the effective capacitance of the array
  $U_{1/2}$ - full width at half maximum (FWHM) of the charging peak, i.e. FWHM of the conductance drop of $\Delta G / G_T$

- Thermometer mode:
  - primary thermometer mode: measure $U_{1/2}$
  - secondary thermometer mode: measure $(\Delta G / G_T)^{-1}$ and determine E_C at a known reference temperature (calibration)

- Interpretation of the thermal decoupling graph:
  The graph shows the CBT electron temperature T_{CBT} (in secondary mode) versus temperature of the bath T_{Cu} (copper in a nuclear refrigerator). Results for two CBTs with different resistances R are shown, with excellent agreement at high temperatures and a full thermal decoupling from T_{Cu} well below 10 mK. Lower temperatures are reached for the high-impedance CBT due to better isolation from the environment by lower dissipation.

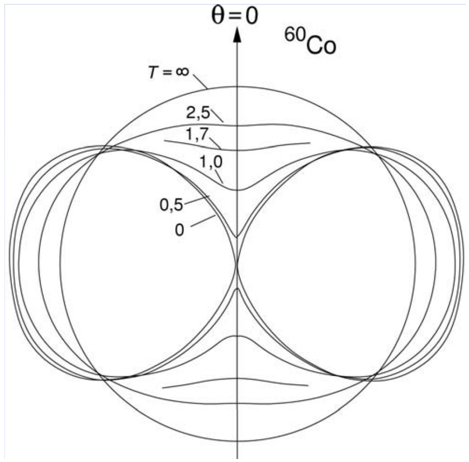
Nuclear Orientation Thermometer

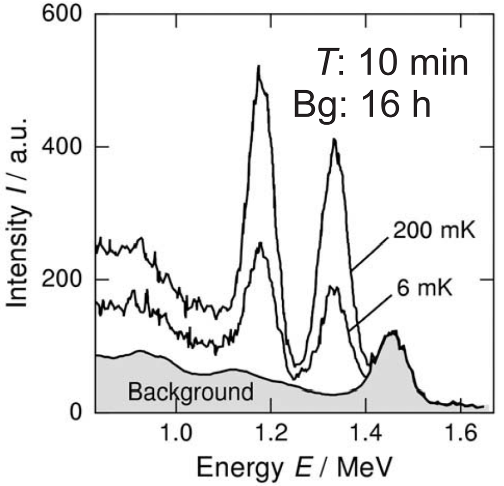
- anisotropic γ radiation from radioactive β decay from nuclei in magnetic field (ferromagnetic matrix)
- probe thermal population of nuclear energy levels

$\Rightarrow$ measure angular distribution of γ radiation

- Problems:
  - heat from radioactive decay completely deposited in the sample
  - long measurement times for low activity samples to obtain acceptable statistical uncertainty
  - discriminate background radiation

$\Rightarrow$ Primary thermometer with small usable temperature range:
^{60}Co in Co matrix: 3 mK ... 30 mK
^{54}Mn in Ni matrix: 6 mK ... 60 mK





Schuster, Hechtfischer, Fellmuth, Rep. Prog. Phys. 57 (1994) 187

- min. uncertainty 0.1% at 10 mK (Marshak, J. Res. NBS **88** (1983) 175)

Noise Thermometry

... based on electronic noise in electric conductors (Johnson noise)

Pros:
- theoretically very well understood
- primary thermometers possible
- extremely large temperature range (... 50 µK ... 2000 K ...)

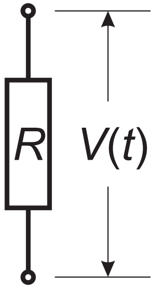
... at low temperatures:
- measurement of noise in thermal equilibrium without excitation
  ⇒ no dissipation
- temperature sensors: metallic, large volume possible
  ⇒ good thermal contact of electrons
- large temperature range (≤50 µK ... 4 K) with single sensor

Cons:
- sufficiently large measurement time (or number of averages) necessary for good statistics:

$$\frac{\Delta T}{T} = \frac{1}{\sqrt{M \cdot N}} = \frac{1}{\sqrt{t_{\mathrm{meas}} \Delta f}}$$

- distinguish or suppress influence of non-thermal noise

Readout Schemes of Noise Thermometers



Thermal voltage fluctuations in resistor *R*: Johnson noise, Nyquist formula

$$\left\langle V^2(t)\right\rangle = S_V \cdot \Delta f \quad \text{with} \quad \boxed{S_V \cong 4k_BTR} \quad \text{for} \quad hf << k_BT$$

Johnson, Phys. Rev. **32**(1928)97,
Nyquist, Phys. Rev. **32**(1928)110

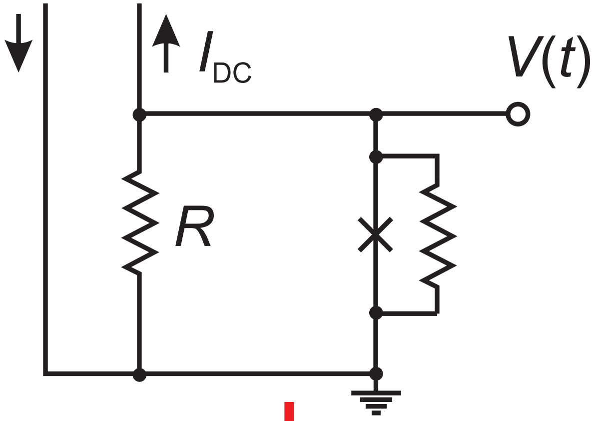
Modulation technique (Basis of the PLTS-2000 at NIST and PTB)
High-frequency carrier generated from Josephson contact („R-SQUID“)
⇒ frequency modulation by thermal noise
Josephson Junction Noise Thermometer (JNT)

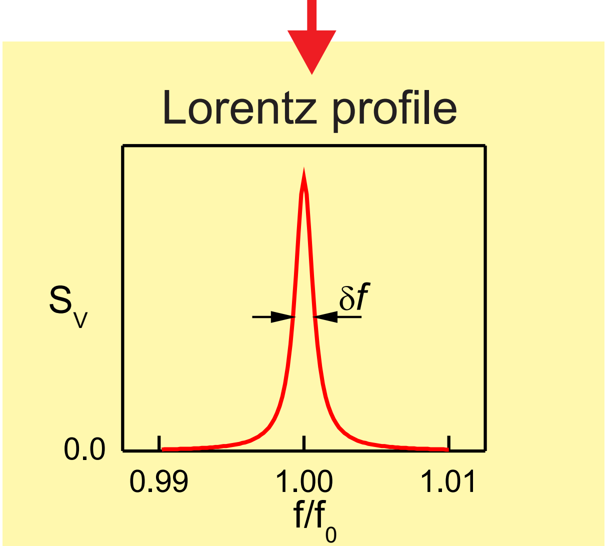
Direct measurement of noise voltage or noise current
⇒ SQUID current sensor as low noise amplifier
Current Sensing Noise Thermometer (CSNT)

Indirect measurement of noise currents
⇒ SQUID magnetometer or gradiometer detecting the corresponding fluctuations of the magnetic field
Magnetic Field Fluctuation Thermometer (MFFT)

Josephson Junction Noise Thermometer

Kamper, Zimmermann, *J. Appl. Phys.* **42** (1971) 132





Josephson effect: $f = V / \Phi_0$

$V = I_{DC} R + V_{noise}$

$\Rightarrow$ frequency modulation by V_{noise} ,
$f_0 = I_{DC} R / \Phi_0$ + side bands

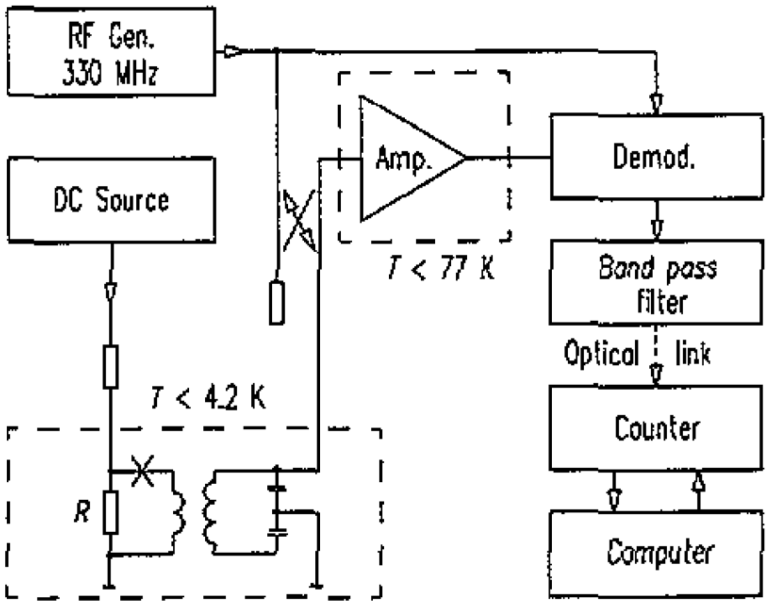
e.g. I_{DC} = 10 mA, R = 10 μΩ $\Rightarrow f_0 \approx$ 50 MHz

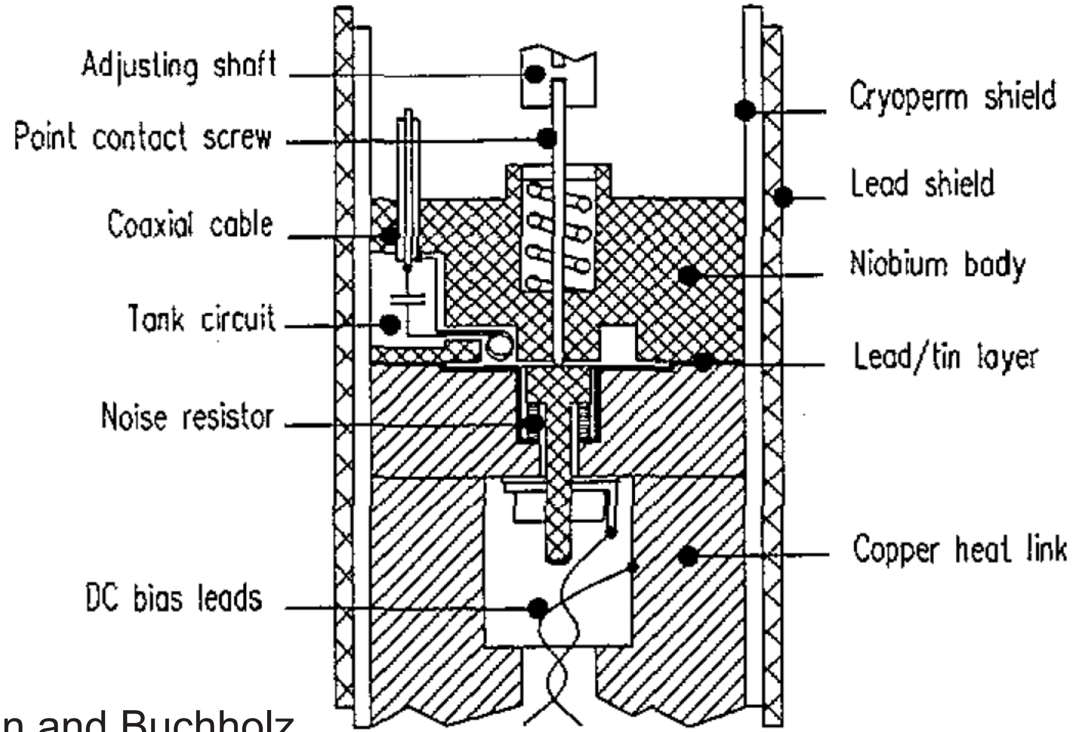
- Primary thermometer

Two measurement techniques:

- Measurement of the spectrum, line width: $$\delta f = \frac{4\pi k_B T R}{\Phi_0^2}$$
- frequency counting, variance at measurement time τ: $$\sigma^2 = \left\langle (f - f_0)^2 \right\rangle = \frac{2 k_B T R}{\tau \Phi_0^2}$$

Josephson Junction Noise Thermometer at PTB





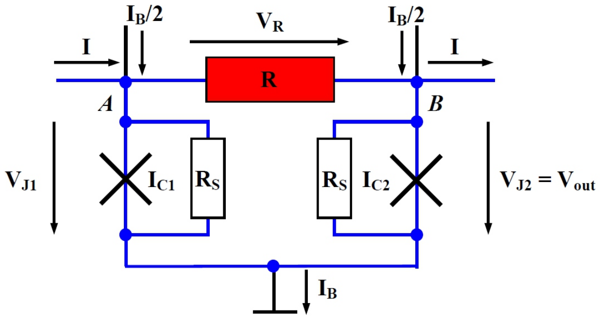
Hoffmann and Buchholz,
J. Phys. E: Sci. Instrum. 17(1984)1035

- uncertainties due to down mixing of external rf noise
- self heating of noise resistor due to bias
- slow: 0.1% (3σ) in a day

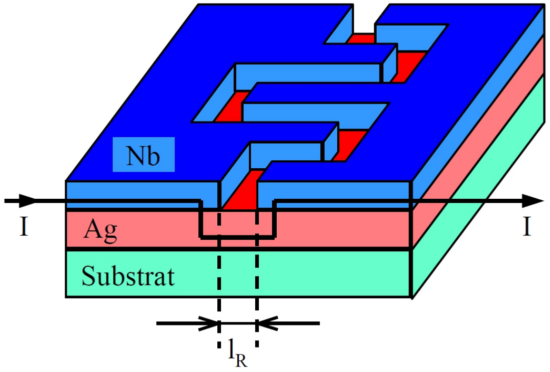
Thin-film DC RSQUID

Stefan Menkel, PhD thesis, 2001: *Integrierte Dünnschicht-dc RSQUIDs für die Rauschthermometrie*

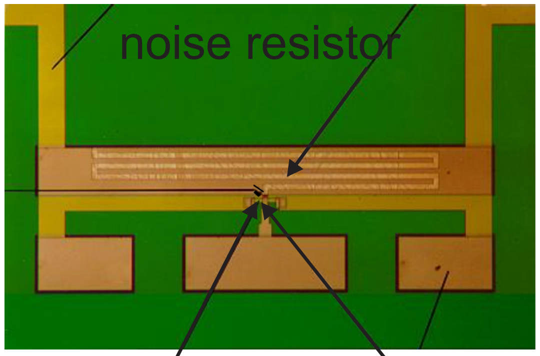
Schematics



Noise resistor

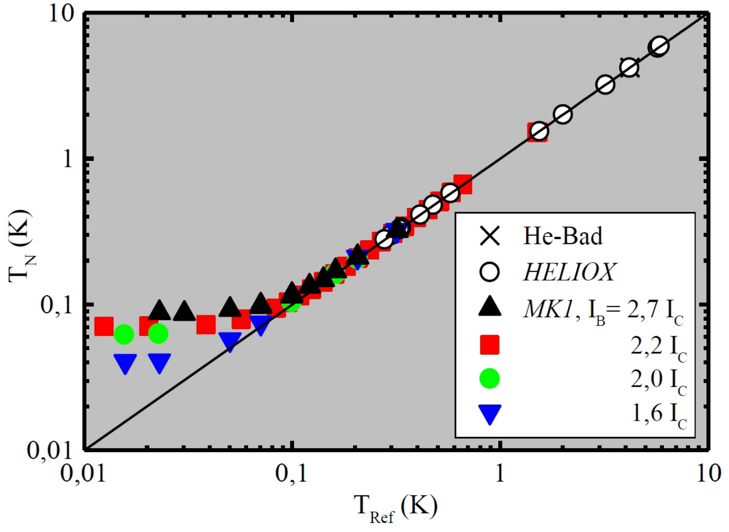


Chip with RSQUID



tunnel junctions shunt

Experimental results



- $R \approx$ (10 ... 20) μΩ
- $\delta T/T$ = 0.66 % in 24 min

- thermal decoupling due to hot-electron effect
- dissipation in shunt resistors

Thin-film DC RSQUID

Background information:

The RSQUID in thin-film technique was thought to replace the bulk-rf RSQUIDs with their practical disadvantages in adjusting the Josephson point contact.
Thin-film devices offer several advantages over bulk devices:

- advantages of the thin-film devices:
  - thin-film tunnel contacts
  - good reproducibility of shape, electrical parameters, ...
  - small devices
  - easy practical use
- dc RSQUID instead of rf RSQUID for easier readout

At the same time a possible disadvantage must be solved or put up with:

- proper integration of the noise resistor into the SQUID loop to achieve good thermal coupling

Results:

- Thermal decoupling below 100 mK for the tested devices.

General:

- Hot-electron effect:
  - Overheating of the electron gas due to the decreasing electron-phonon coupling at low temperatures. The electron system can have considerably higher temperatures than the phonons and/or completely decouple from the phonon temperature.
  - Cf. F.C. Wellstood, C. Urbina, and J. Clarke, Hot-electron effects in metals, Phys. Rev. B **49** (1994) 5942
  - Compare with results found for the CBTs.

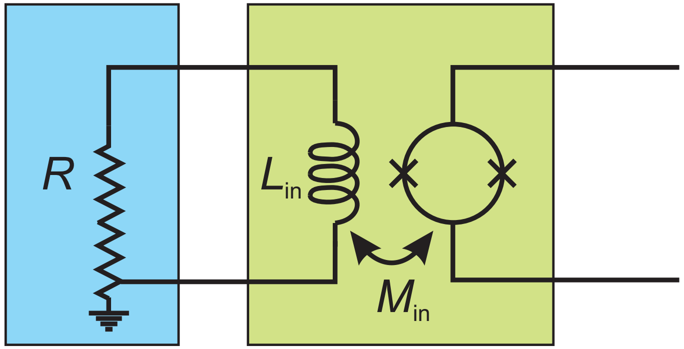
CSNT and MFFT

CSNT

Current Sensing Noise Thermometer

Noise Resistor

SQUID Sensor



Variable T

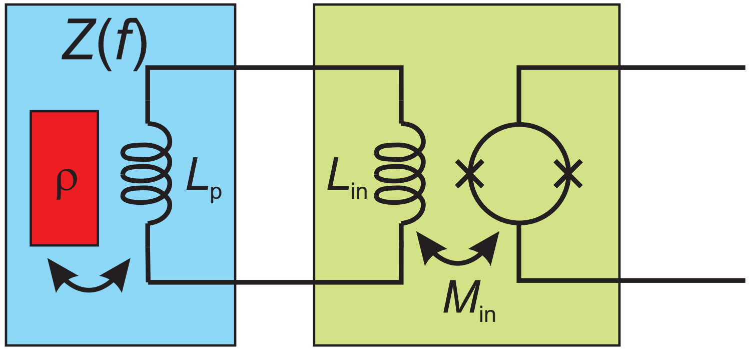
Fixed T
or Variable T

MFFT

Magnetic Field Fluctuation Thermometer

Noise Source *

SQUID Sensor *



Variable T

Fixed Temperature
or Variable T

Secondary thermometer

$R(T)$ = const.

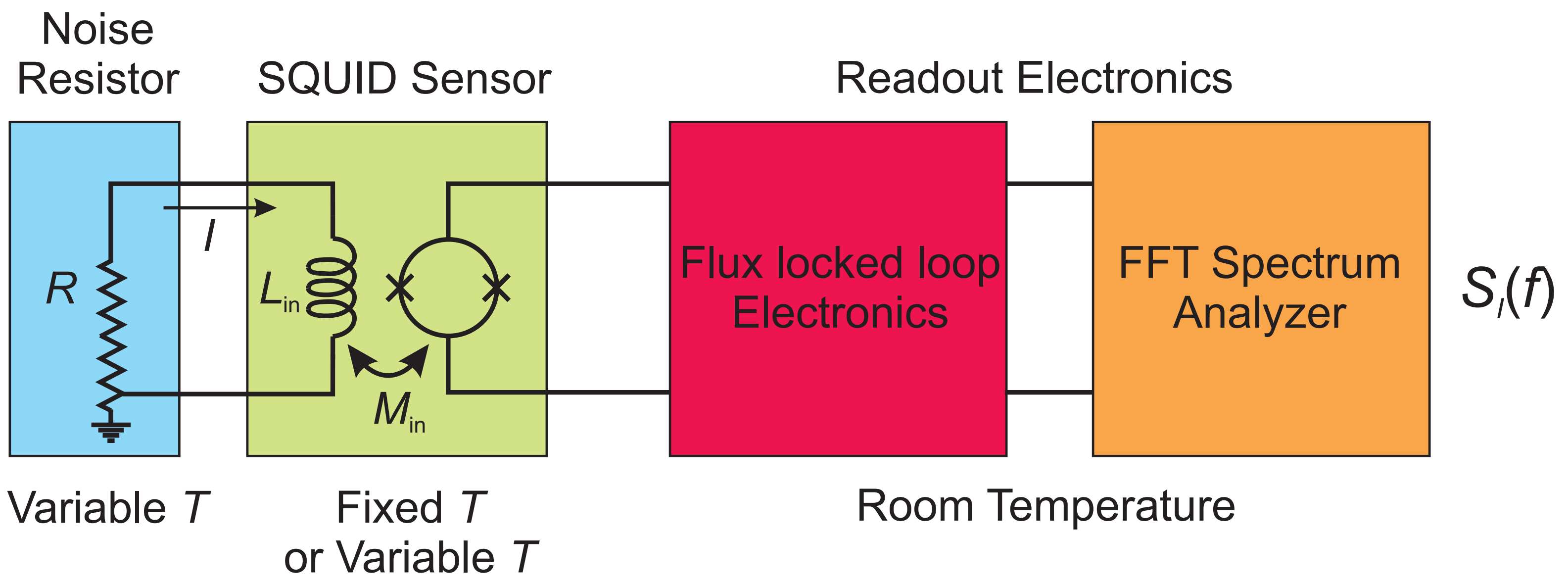
$$\frac{T}{T_{\mathrm{Ref}}} = \frac{S_x(f,T)}{S_x(f,T_{\mathrm{Ref}})}$$

$\rho(T)$ = const.
$\mu(T)$ = const.

* alternatively for MFFT:
multiloop SQUID field sensor

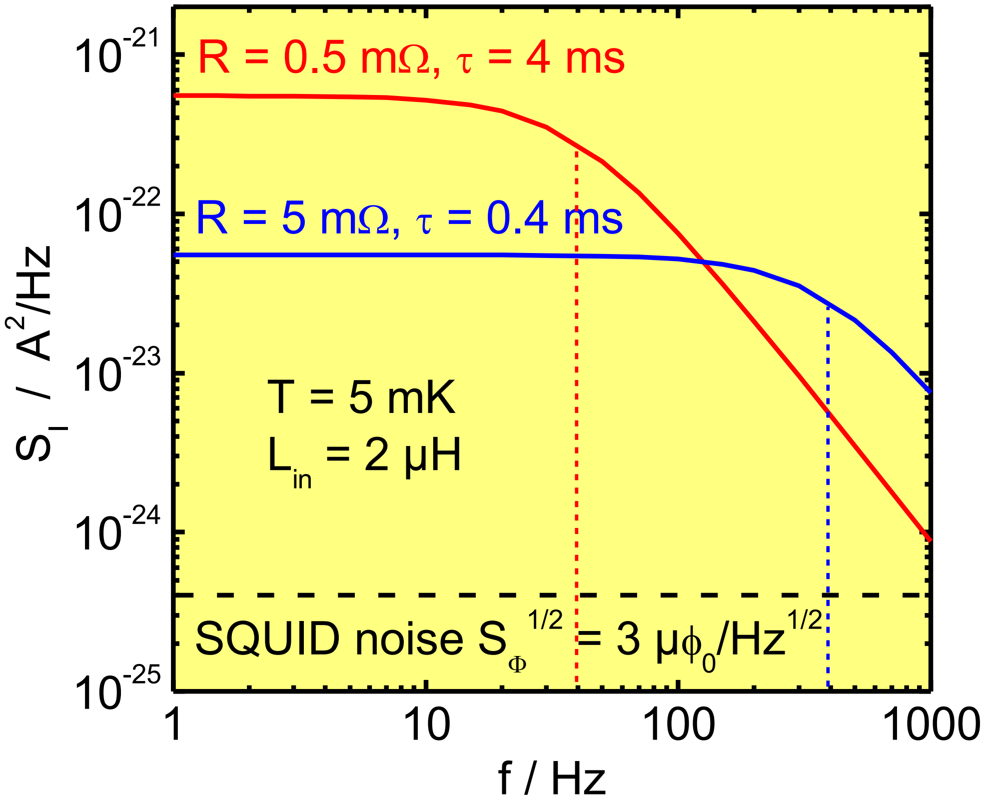
Current Sensing Noise Thermometer (CSNT)

⇒ direct measurement of noise current $I(t)$ and its spectral density $S_I(f)$



L-*R* circuit: first-order low pass

CSNT: Spectral Density S_I



L-*R* circuit: first-order low pass

$$S_I(f) = \frac{4k_BT}{\boxed{R}}\left(\frac{1}{1+(f/f_c)^2}\right)$$

$$f_c = \frac{\boxed{R}}{2\pi L}$$

total energy: $\int_0^\infty \frac{L}{2} S_I(f)\mathrm{d}f = \frac{1}{2}k_\mathrm{B}T$ ⇒ average magnetic energy stored in the coil (inductivity L) as expected for single degree of freedom

energy below f_c: $\frac{1}{4}k_BT$ ⇒ 50% for low pass of first order

CSNT Performance

SQUID with coupled energy resolution ε_C:
$$\varepsilon_C = \frac{1}{2} L_{\text{in}} S_I = \frac{1}{2} L_{\text{in}} \frac{S_\Phi}{M_{\text{in}}^2}$$

CSNT noise temperature:
$$T_N = \left(\frac{\varepsilon_c}{2k_B}\right)\left(\frac{R}{L}\right)$$

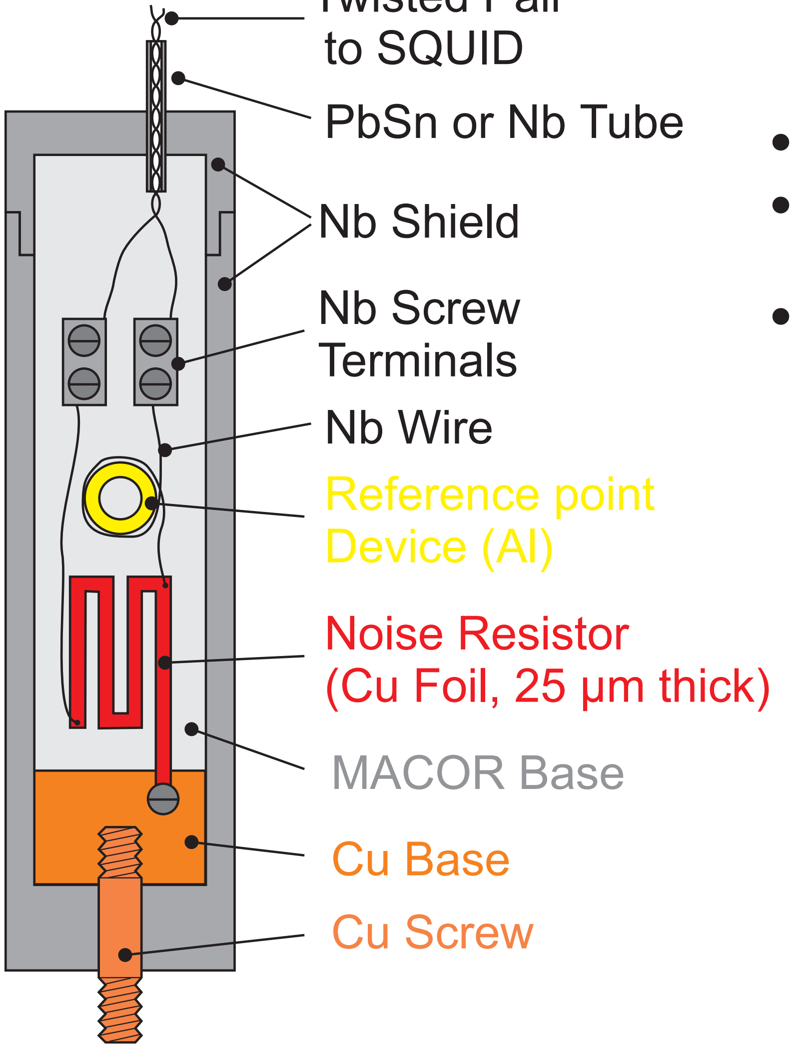
Speed of the thermometer:
$$\sigma = \frac{\Delta T}{T} \approx \left(\frac{2\tau}{t_{\text{meas}}}\right)^{1/2} = \left(\frac{2L}{t_{\text{meas}} R}\right)^{1/2}$$

Figure of merit: $T_N \sigma^2 t_{\text{meas}} = \frac{\varepsilon_C}{k_B}$

⇒ only determined by SQUID sensor

M.L. Roukes, R.S. Germain, M.R. Freeman, R.C. Richardson,
DC SQUID Noise Thermometry , LT-17 Proceedings, 1177 (1984)

CSNTs (I)



Twisted Pair to SQUID

PbSn or Nb Tube

Nb Shield

Nb Screw Terminals

Nb Wire

Reference point Device (Al)

Noise Resistor (Cu Foil, 25 µm thick)

MACOR Base

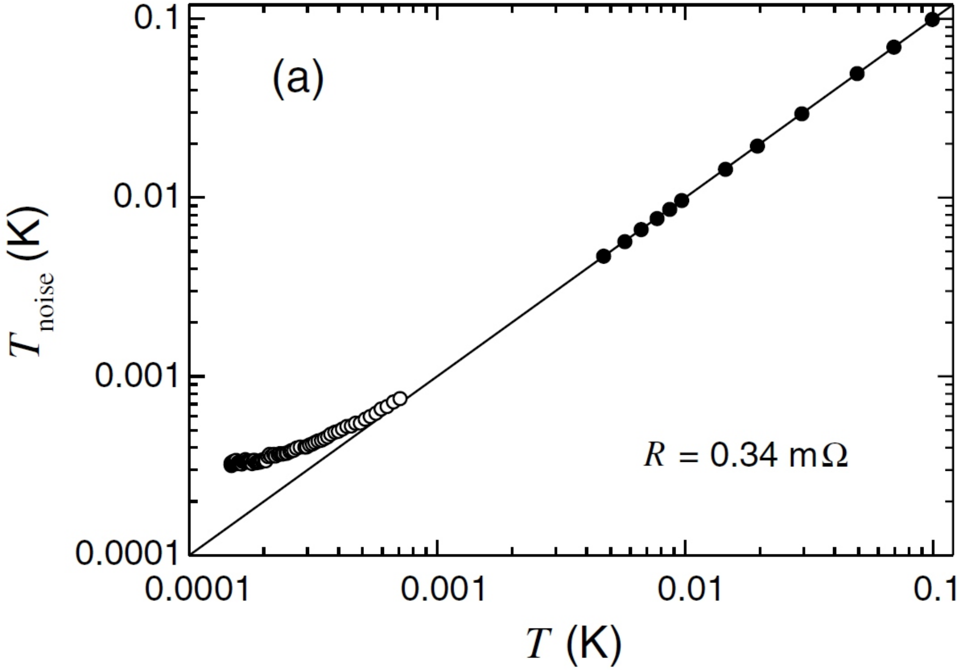
Cu Base

Cu Screw

Lusher et al, Meas. Sci. Technol. 12 (2001) 1

Royal Holloway University of London

- T_N = 1.8 µK, $f_c \approx$ 24 Hz
- T_{min} = 0.3 mK due to temperature independent heat leak
- precision of 1% in 145 s



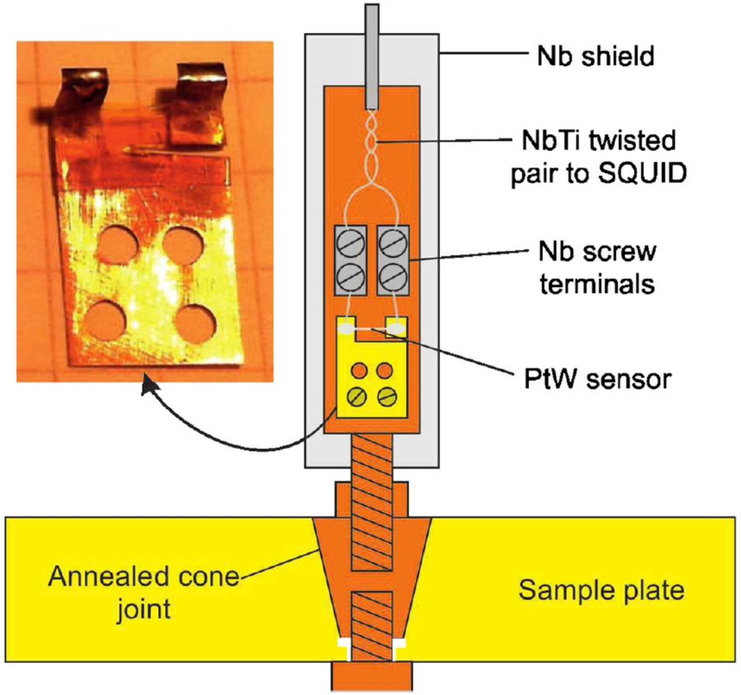
CSNTs (II)

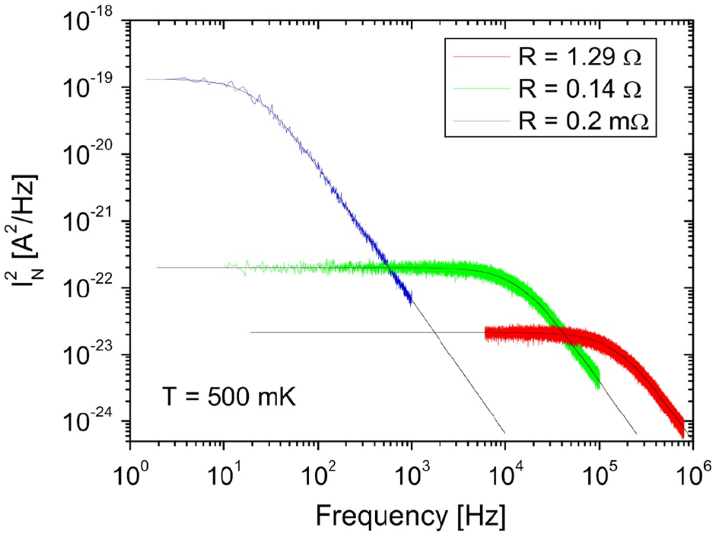
Royal Holloway University of London

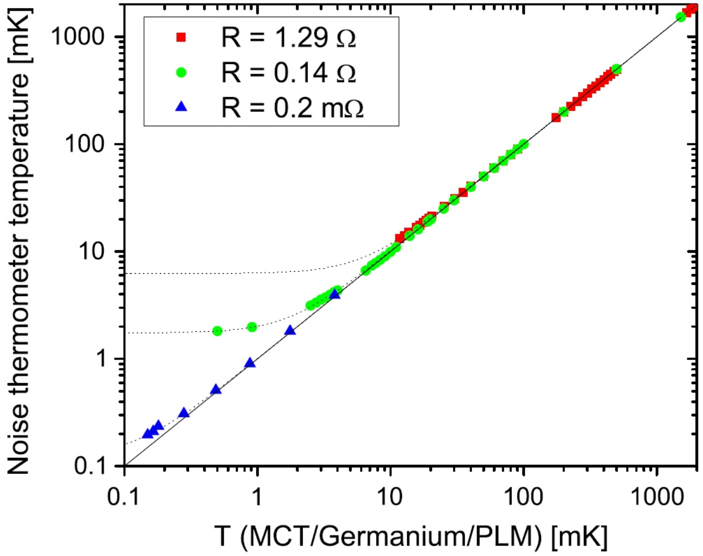
PTB

Casey et al, J Low Temp Phys 175 (2014) 764:

Current Sensing Noise Thermometry: A Fast Practical Solution to Low Temperature Measurement







CSNTs (III)

Comment on Casey’s work:

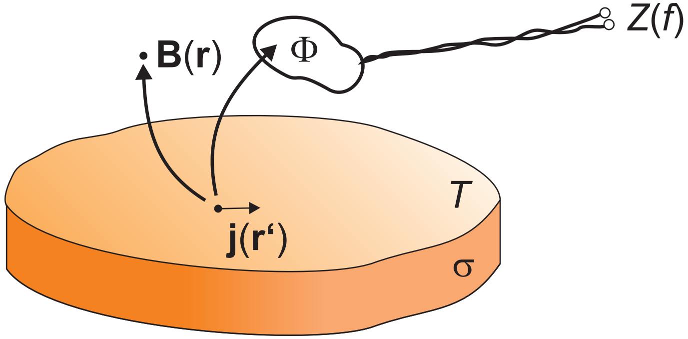
- New implementation of former CSNTs developed at the Royal Holloway University of London, making the CSNT a fast and practical thermometer that can be used as standard thermometer.

- Better adaption to experimental conditions (temperature range) by choosing the optimal noise sensor from a wide range of resistances.
  ⇒ This allows either optimization for speed (limited to higher temperatures)
  or optimization for low noise temperatures (at the cost of speed).

Magnetic Field Fluctuation Thermometer (MFFT)

Idea: *A. Fleischmann, Universität Heidelberg, 2002* *

⇒ detect the thermal magnetic flux noise caused by thermally activated noise currents **j**(**r**') in a conductor at *T* by means of a SQUID sensor



Power Spectral Density (PSD)

$$S_\Phi(f,T) = \frac{4k_B T \mathrm{Re}(Z(f))}{(2\pi f)^2}$$

Different implementations:

- wire-wound coil on temperature sensor, connected to (distant) SQUID current sensor
- integrated multiloop SQUID gradiometer directly above the surface of the temperature sensor

Thermally robust:

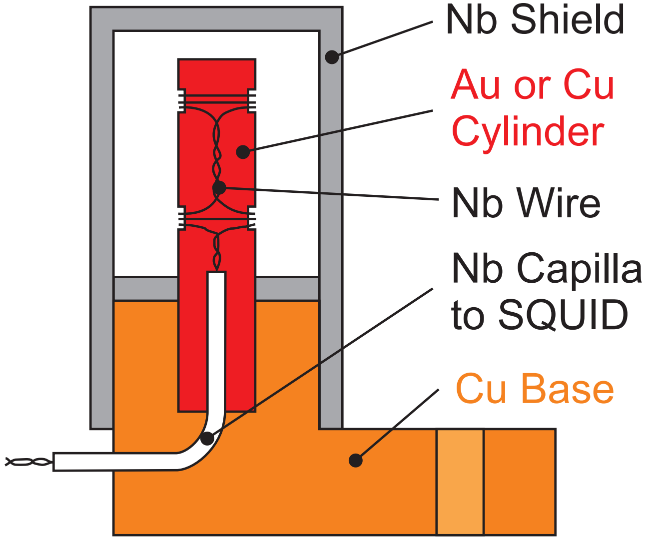
- good thermal contact of a massive temperature sensor
- large volume for electron-phonon coupling

*Private communication, 2005. First paper on first implementation: A. Netsch et al, AIP **850** (2006) 1593

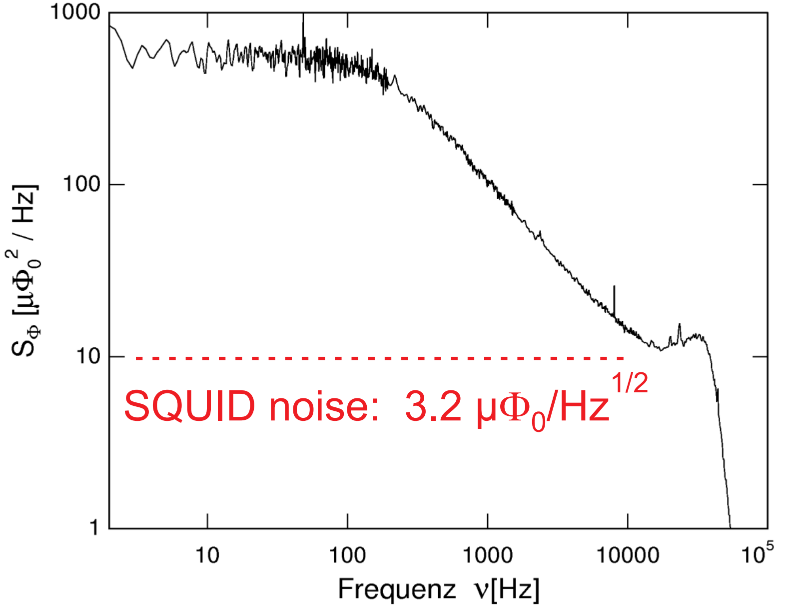
MFFTs (I)

A. Netsch et al, AIP 850 (2006) 1593

$\Rightarrow$ axial, wire-wound gradiometer around cylinder



T = 13 mK, Au (RRR = 110)



$\Delta T/T \cong 0.7\%$ in 25 s (Δf = 500 Hz)

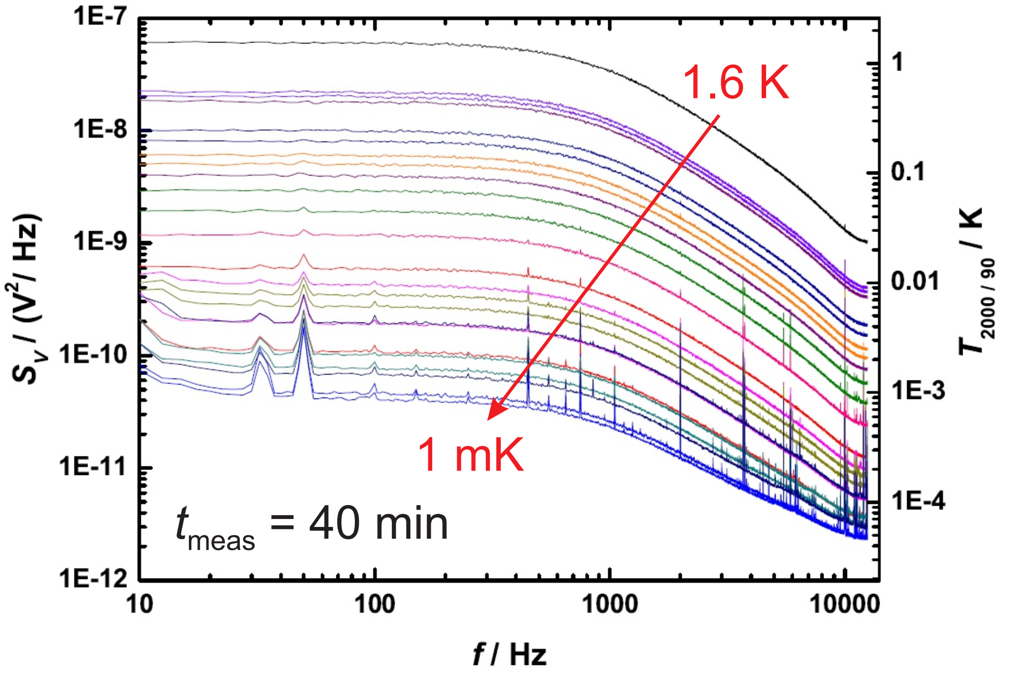
noise sources:

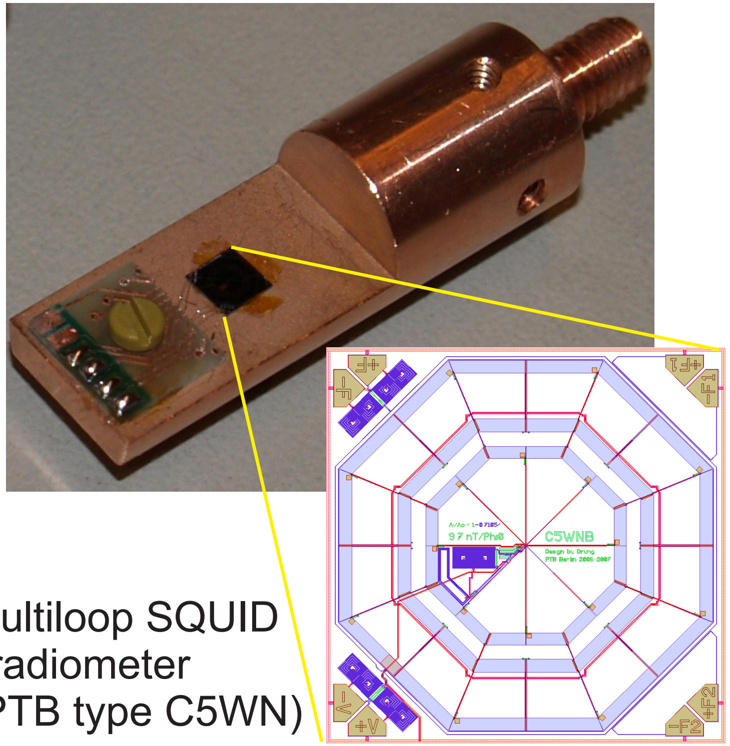
- Au (>99.999%) , ∅ 2 mm, RRR = 110
- Cu (>99.999%) , ∅ 2.5 mm, RRR = 1000

MFFTs (II)

MAGNICON
physical research and instrumentation

Engert et al, Int J Thermophys **28** (2007) 1800
Beyer et al, Supercond. Sci. Technol. **26** (2013) 065010



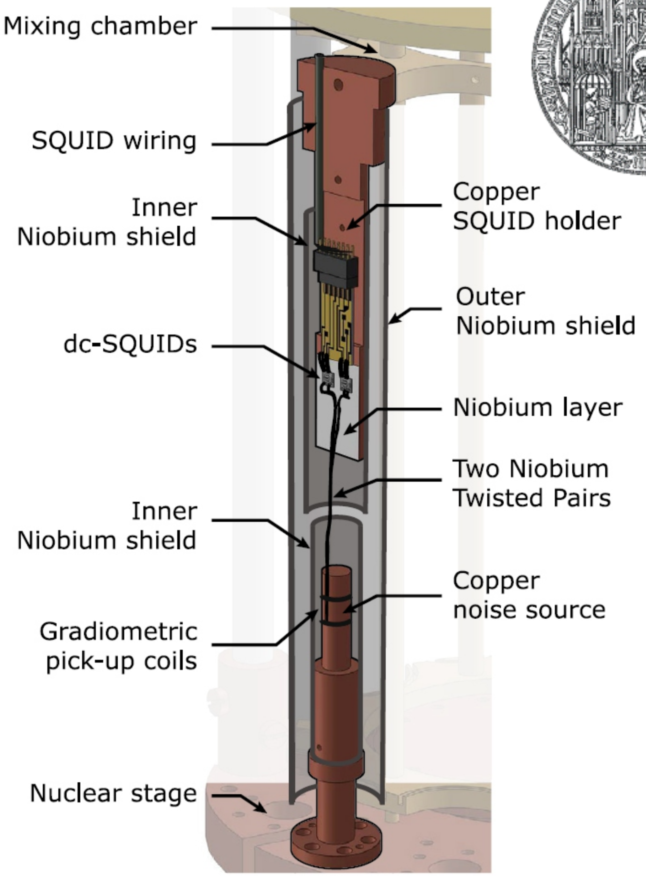


multiloop SQUID
gradiometer
(PTB type C5WN)

MFFTs (III)

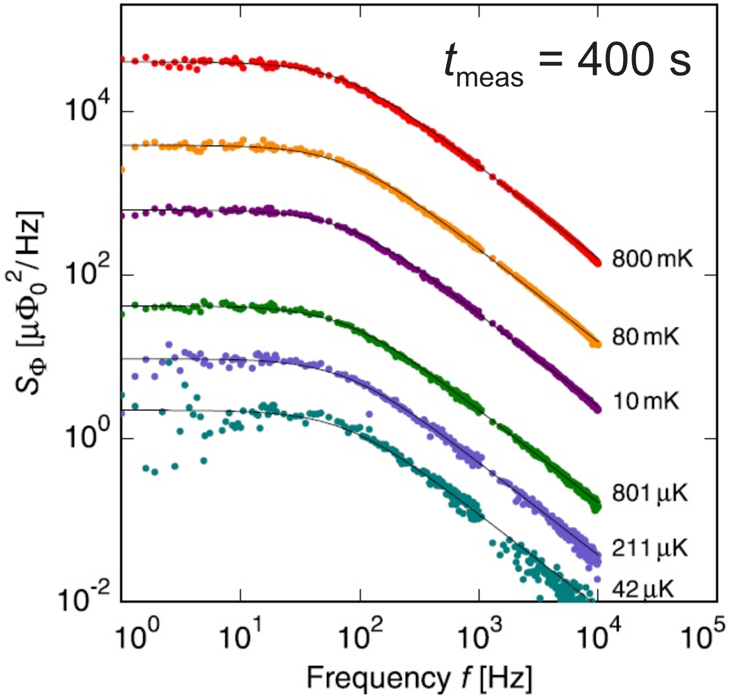
Cross-correlation technique to suppress non-thermal noise (e.g. SQUID noise)

⇒ cross-correlation of two independent channels measuring the same signal



Rothfuß et al, J Low Temp Phys 175 (2014) 776

Cross-power spectral density



⇒ resolution enhancement of ≥15 @ 45 µK

Conclusion

- further increasing number of primary thermometers (primary versions of CSNT & MFFT in development)
- weakness of thin-film devices: thermal decoupling at low temperatures due to the hot-electron effect
- active development of SQUID based thermometers (now and in the last decade)
- variety of noise thermometers: CSNT, MFFT, ...
- cross-correlation technique enhances resolution of noise thermometers (MFFT)

- ***Mise en pratique*** for the definition of the kelvin (MeP-K)
  http://www.bipm.org/en/publications/mep_kelvin/

  Scope:
  „This document provides the information needed to perform a practical measurement of temperature in accord with the International System of Units (SI).“