IEEE/CSC & ESAS SUPERCONDUCTIVITY NEWS FORUM (global edition), October 2015.
Invited keynote presentation 3A-LS-O1.3 given at EUCAS 2015; Lyon, France, September 6 – 10, 2015.
Not submitted to *IEEE Trans. Appl. Supercond.*

Recent Developments in Finite Element Methods for Electromagnetic Problems

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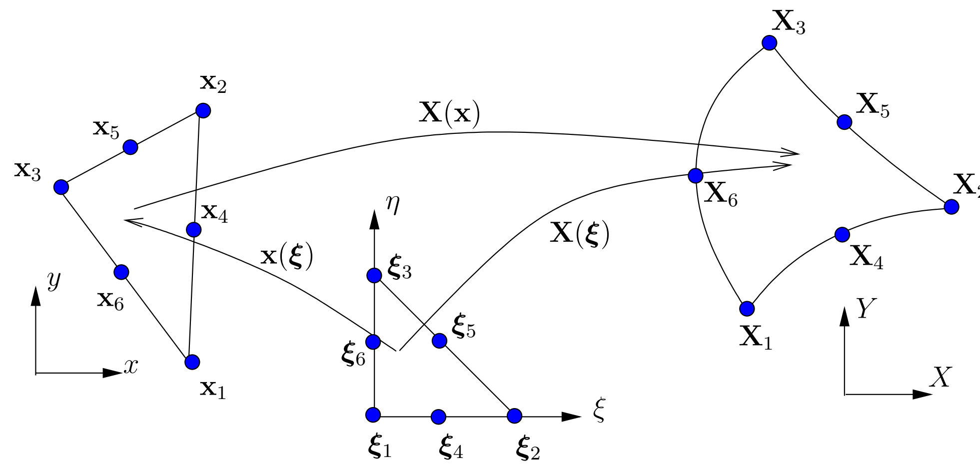
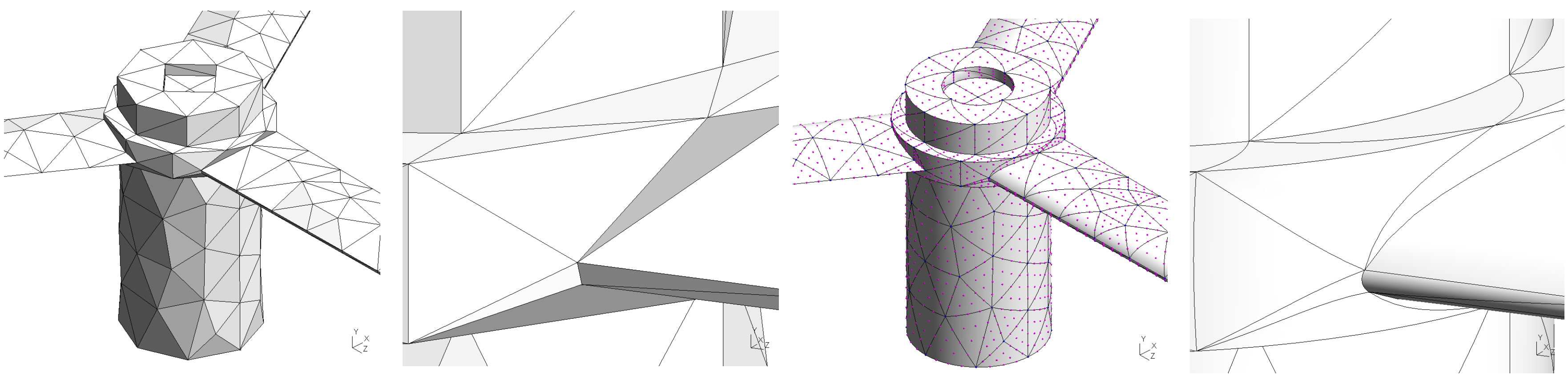
A (subjective) selection

- High-order and isogeometric methods — *for fast convergence*
- Domain decomposition techniques — *for large scale computations*
- Multiscale methods — *for microstructured, hysteretic materials*
- Stochastic solvers — *for material/geometric uncertainties*
- Homology & cohomology solvers — *for field-potential problems*
- …

- High-order and isogeometric methods
- Domain decomposition techniques
- Multiscale methods
- Stochastic solvers
- Homology & cohomology solvers

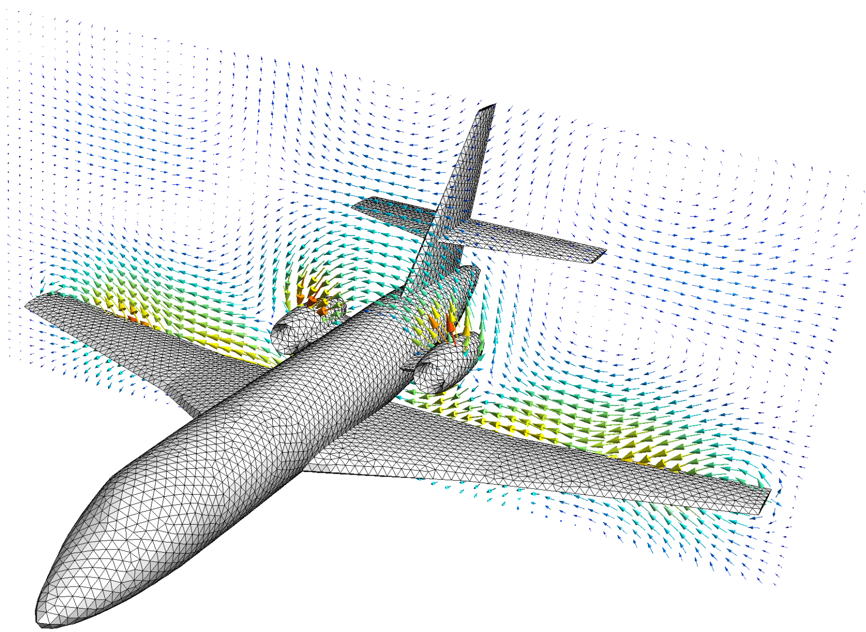
- High-order polynomial approximations: curved mesh generation; provable validity
- Isogeometric analysis: directly based on CAD (e.g. T-splines)

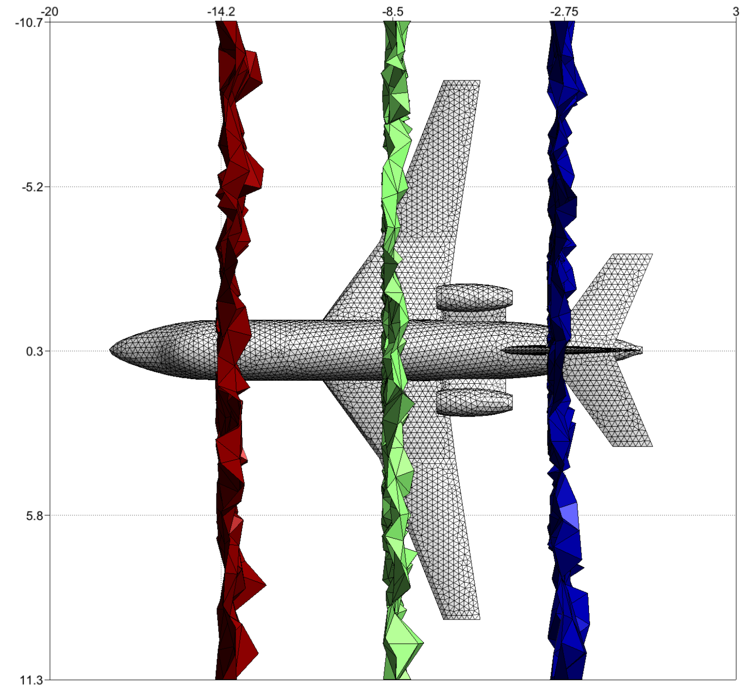




- High-order and isogeometric methods
- Domain decomposition techniques
- Multiscale methods
- Stochastic solvers
- Homology & cohomology solvers

- Split computational domain into subdomains, and iterate
- Scalability requires “coarse grid” (long-distance information exchange); open problem for high-frequency wave propagation





- Time-domain approaches gaining traction (Waveform relaxation, Parareal, …)

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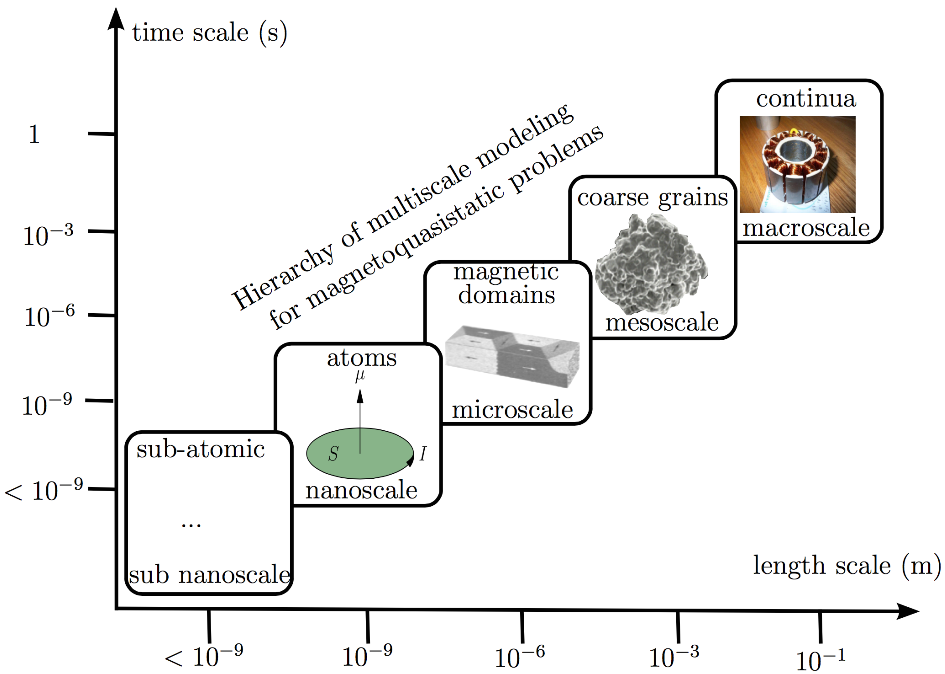
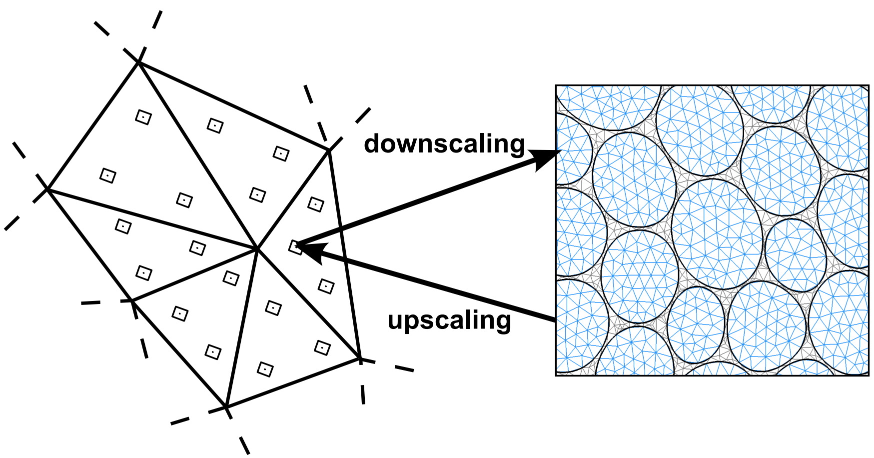
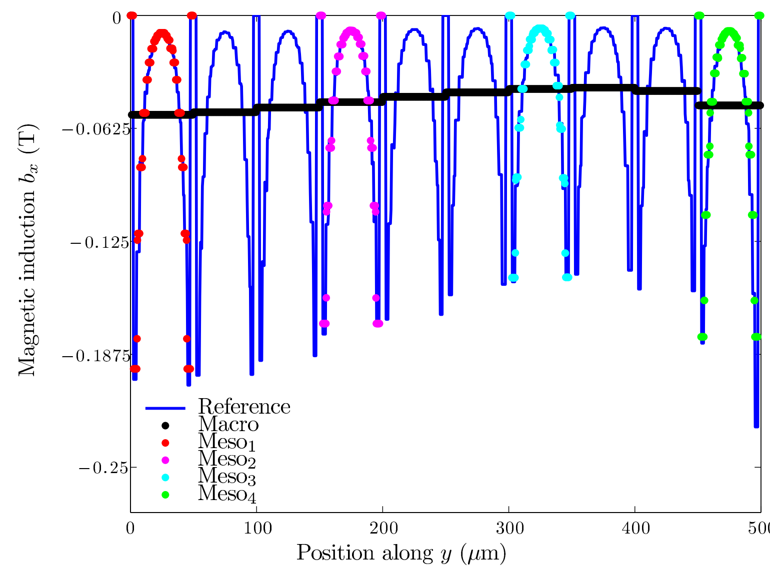
- High-order and isogeometric methods
- Domain decomposition techniques
- **Multiscale methods**
- Stochastic solvers
- Homology & cohomology solvers



- Modern computer architectures enable fully computational homogenization
- Possibly heterogeneous models
- Convergence of local (in the microstructure) and global quantities





- High-order and isogeometric methods
- Domain decomposition techniques
- Multiscale methods
- **Stochastic solvers**
- Homology & cohomology solvers

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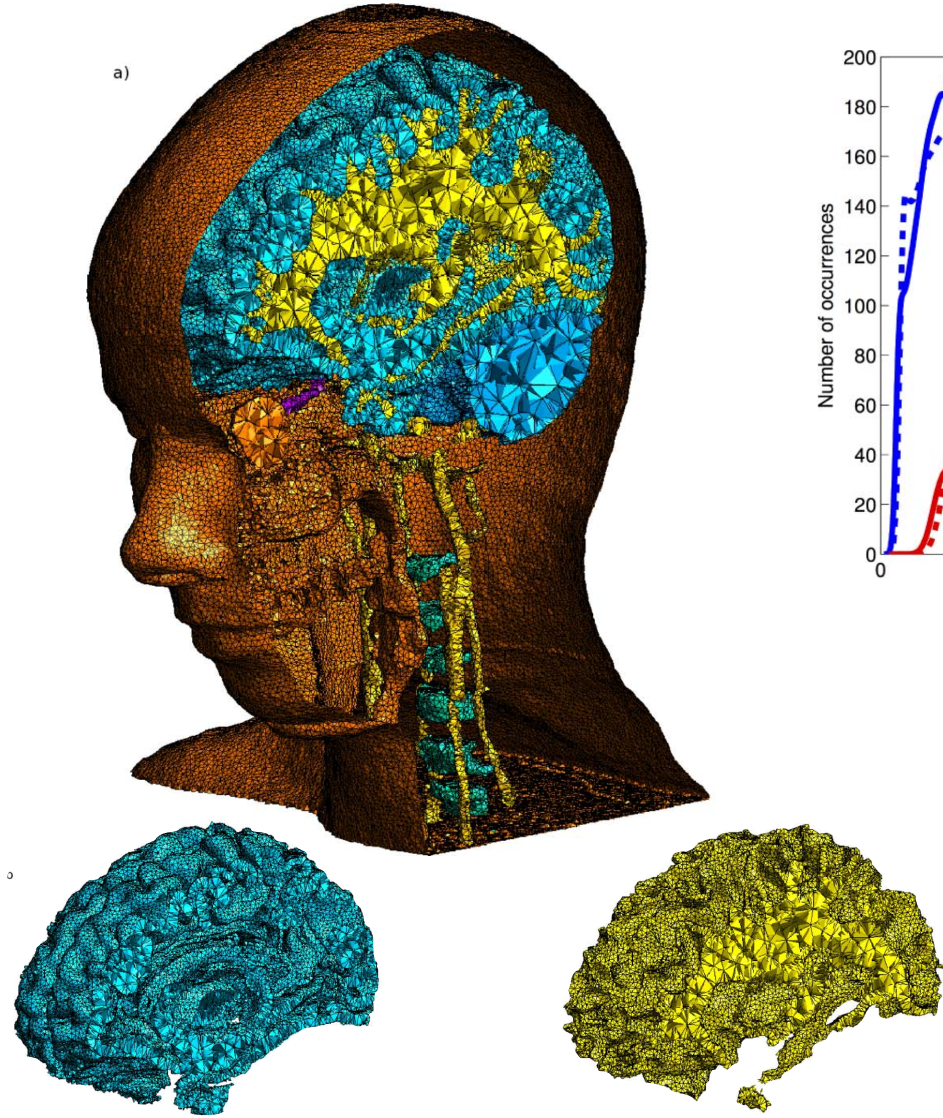
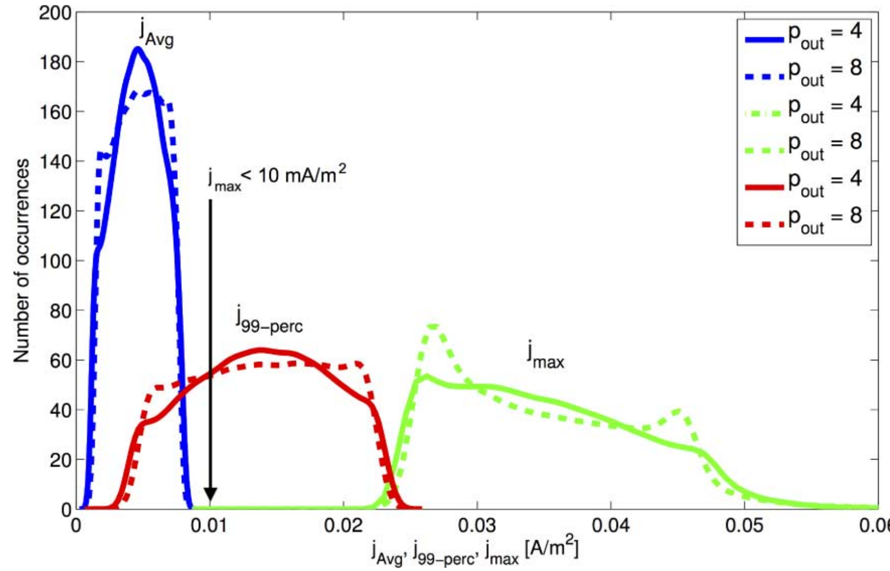


$\sigma_G(\omega) \sim \mathrm{U}([0.0753;\ 0.5155])\ (\mathrm{S/m})$ $\sigma_W(\omega) \sim \mathrm{U}([0.0533;\ 0.3020])\ (\mathrm{S/m})$



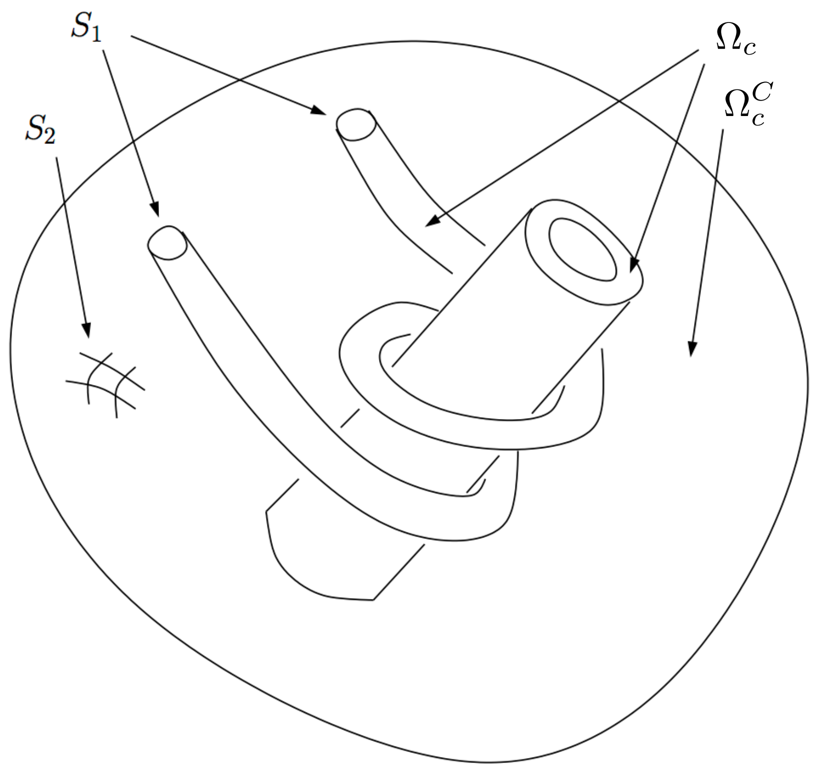
- Uncertain geometry and/or material laws
- Efficient calculation of probability distribution of quantities of interest

- High-order and isogeometric methods
- Domain decomposition techniques
- Multiscale methods
- Stochastic solvers
- Homology & cohomology solvers

Homology & cohomology solvers

Consider the model problem of an inductor around a hollow piece of conductor:

- Conducting domain Ω_c
- Non-conducting domain Ω_c^C
- Overall domain $\Omega = \Omega_c \cup \Omega_c^C$
- Complementary parts of boundary of Ω : S_1 and S_2



Homology & cohomology solvers

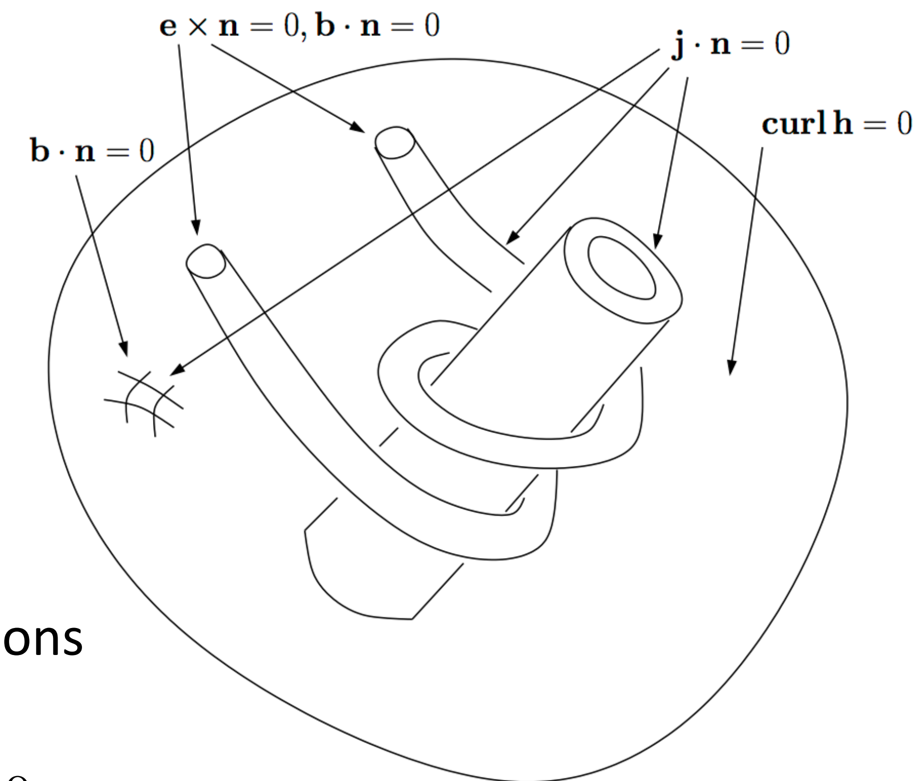
- In Ω we want to solve

$$\mathbf{curl}\,\mathbf{h} = \mathbf{j}$$
$$\mathbf{curl}\,\mathbf{e} = -\partial_t \mathbf{b}$$
$$\mathrm{div}\,\mathbf{b} = 0$$

together with constitutive laws

$$\mathbf{b} = \mu\,\mathbf{h}$$
$$\mathbf{e} = \rho\,\mathbf{j}$$

and appropriate boundary conditions



- In Ω_c^C, $\mathbf{j} = 0$ and thus $\mathbf{curl}\,\mathbf{h} = 0$

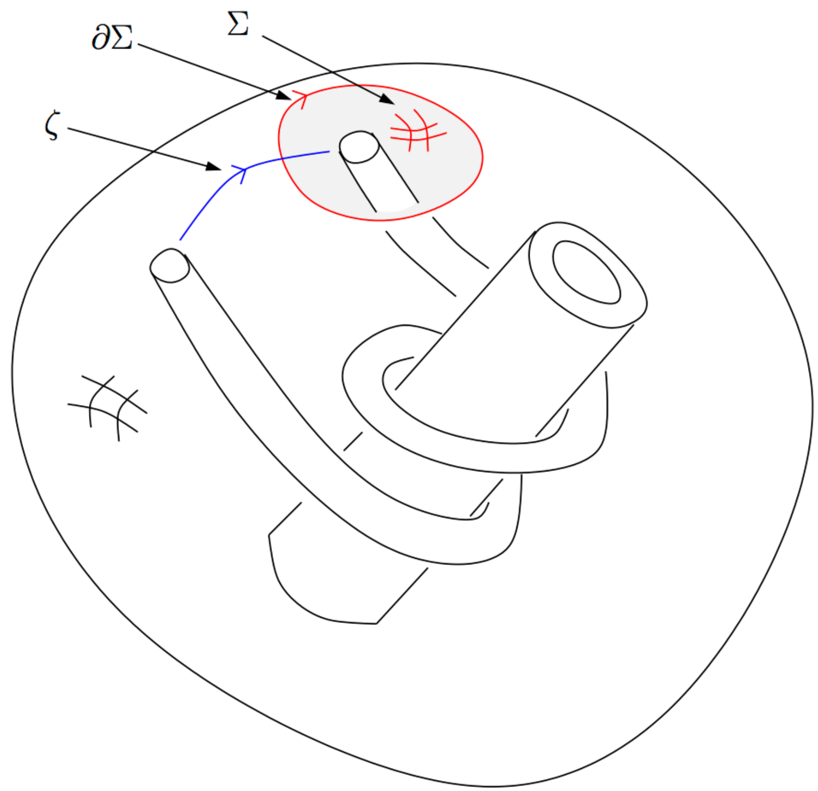
Homology & cohomology solvers

We can drive the problem either by the net current through the terminals

$$\int_\Sigma \mathbf{j}\cdot\mathbf{n}\,\mathrm{d}a = \int_\Sigma \mathbf{curl}\,\mathbf{h}\cdot\mathbf{n}\,\mathrm{d}a = \int_{\partial\Sigma} \mathbf{h}\cdot\mathrm{d}\mathbf{l} = I_s$$

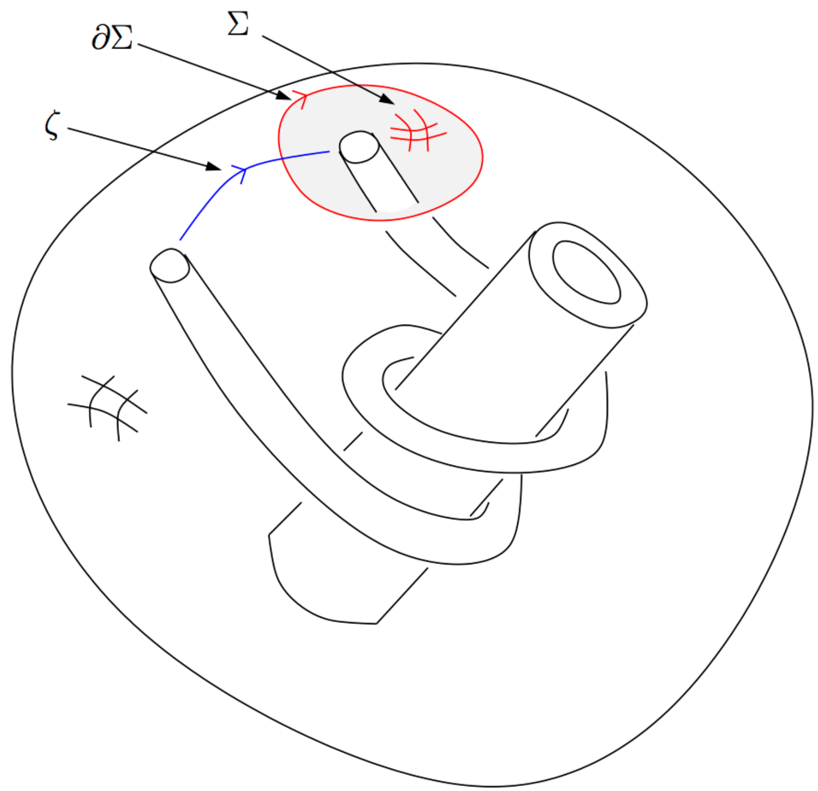
or by the net voltage difference between the terminals

$$\int_\zeta \mathbf{e}\cdot\mathrm{d}\mathbf{l} = V_s$$



Homology & cohomology solvers

- The curves $[\zeta]$ belong to the relative homology space $H_1(S_2, \partial S_2)$
- The curves $[\partial\Sigma]$ belong to the homology space $H_1(S_2)$
- The integral conditions on fields $\mathbf{e}$ and $\mathbf{h}$ fixing V_s and I_s are called cohomology conditions (they consider integrals over an element of a homology space)



Homology & cohomology solvers

- Finite element H-formulation: look for the magnetic field $\mathbf{h}$ such that

$$\partial_t \int_{\Omega} \mu\, \mathbf{h} \cdot \mathbf{h}' \,\mathrm{d}V + \int_{\Omega_c} \rho\, \mathbf{curl}\, \mathbf{h} \cdot \mathbf{curl}\, \mathbf{h}' \,\mathrm{d}V + \int_{S_2} \mathbf{e} \times \mathbf{h}' \cdot \mathbf{n} \,\mathrm{d}a = 0$$

holds for appropriate test functions $\mathbf{h}'$

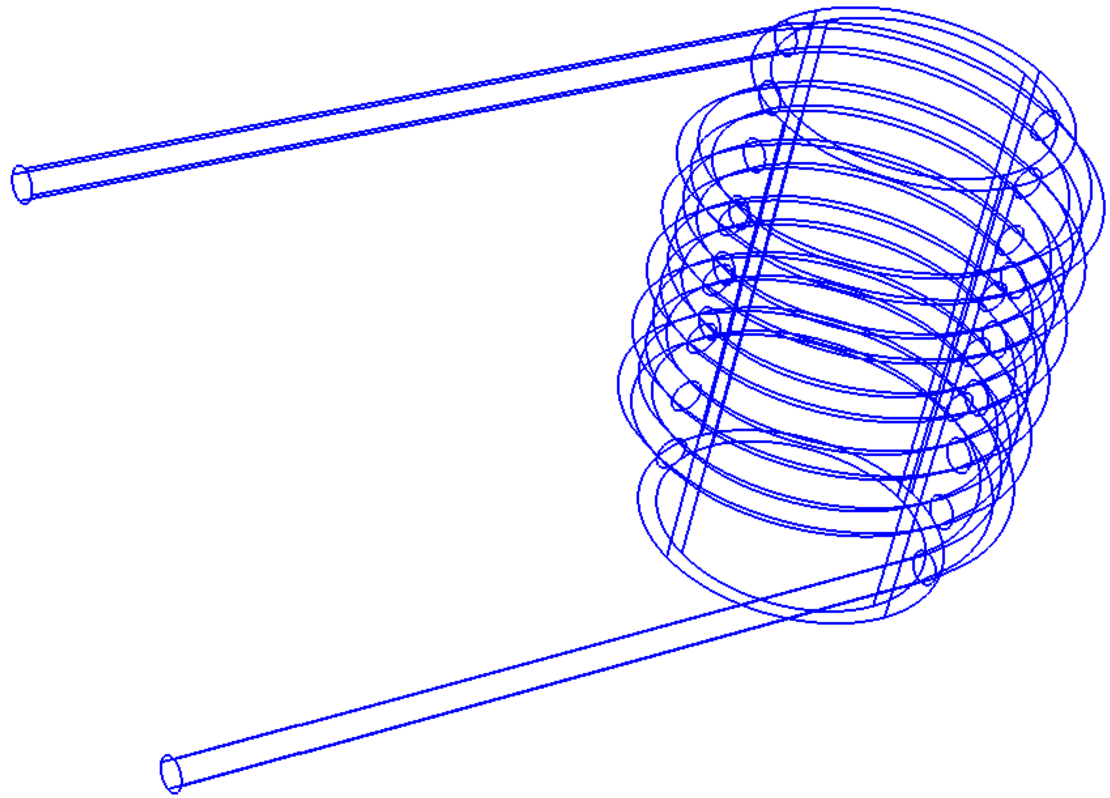
- At the discrete level, if N and E denote the sets of nodes and edges on in the mesh, and n_i and e_i the associated nodal and edge shape functions:

$$\mathbf{h} = \sum_{i \in N(\Omega_c^C)} \phi_i\, \mathbf{grad}\, \mathsf{n}_i + \sum_{i \in E(\Omega_c \setminus (\partial\Omega_c^C \cap \partial\Omega_c))} h_i\, \mathsf{e}_i + I_1 \mathsf{E}_1 + I_2 \mathsf{E}_2$$

where E_i are the (discrete) cohomology basis functions of $H_1(\Omega_c^C)$

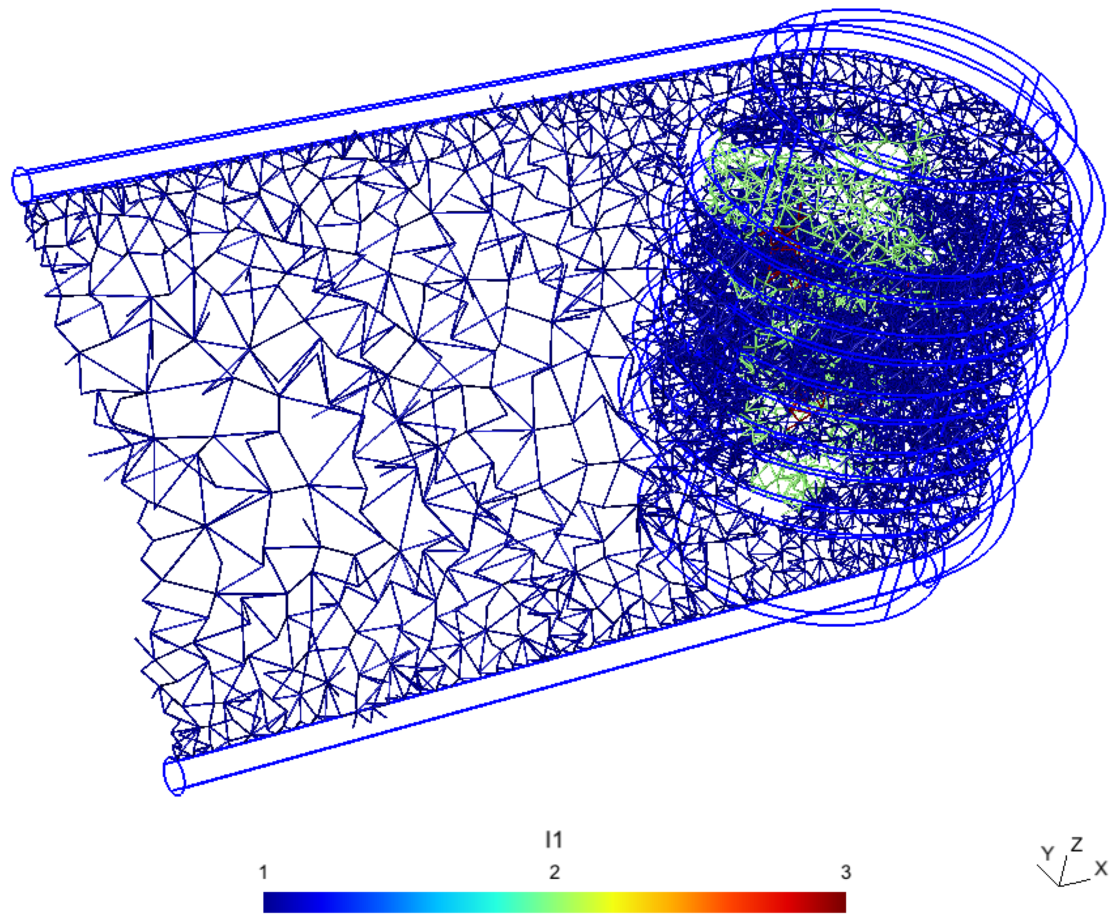
Homology & cohomology solvers

In practice:



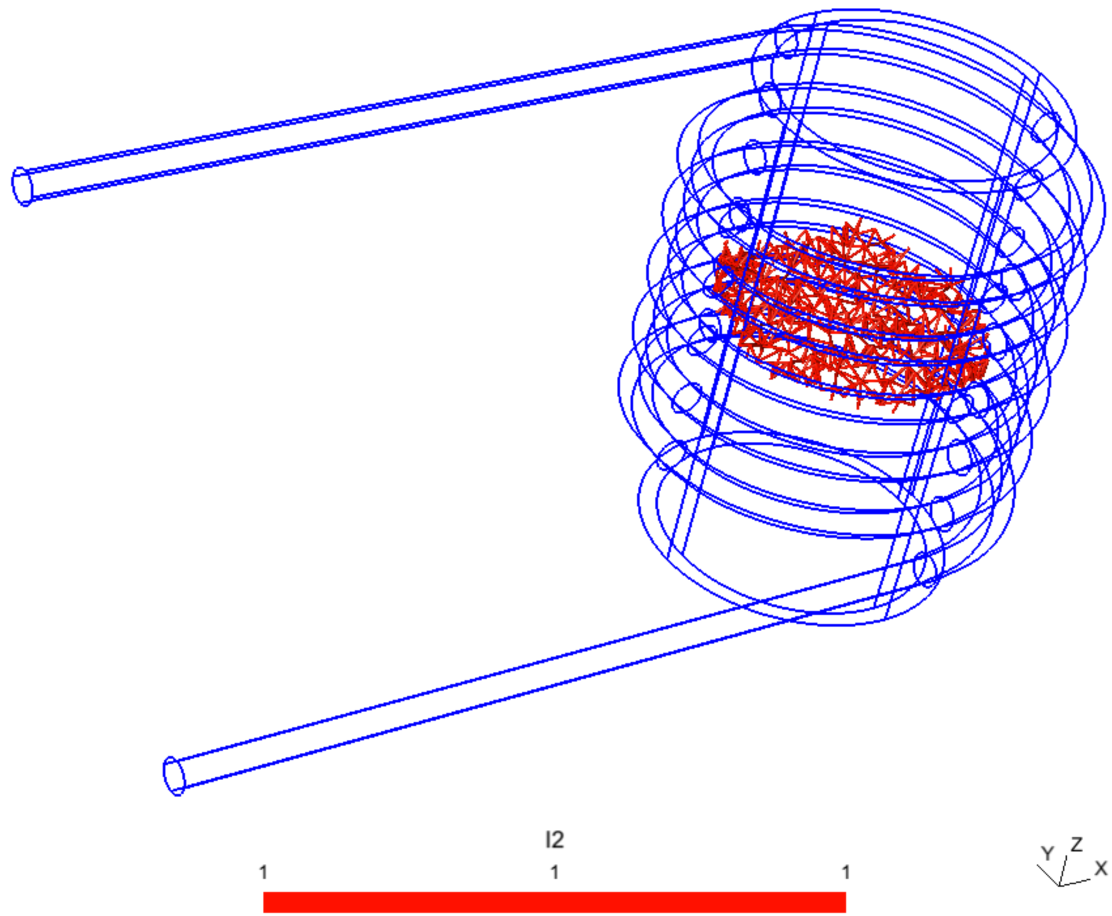
Homology & cohomology solvers

$$\mathsf{E}_1 = \sum_j z_1^j \mathsf{e}_j$$



Homology & cohomology solvers

$$\mathsf{E}_2 = \sum_j z_2^j \mathsf{e}_j$$



Example: superconducting wire

- Parametric 3D geometry (in “helix.geo” file):
  - Number and layers of superconducting filaments
  - Twist pitch, radius of surrounding air box, conducting matrix and filaments
- Nonlinear H-formulation, with imposed time-varying (total) current (in “helix.pro” file):
  - No fictitious conductivity in the air, current imposed through cohomology basis function
  - Nonlinear resistivity: $\rho = \frac{E_c}{J_c}(\frac{\|\boldsymbol{j}\|}{J_c})^{n-1}$
  - Implicit Euler time-stepping

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Example: superconducting wire

- You can give it a try now:
  - Download the code from http://onelab.info
  - Uncompress the archive and launch Gmsh (icon)
  - Open the file "models/superconductors/helix.pro" with the "File>Open" menu
  - Change some parameters and click on "Run"
- The code combines the mesh generator Gmsh (http://gmsh.info) and the finite element solver GetDP (http://getdp.info). Both are free software, released under GNU GPL.

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Demo!

Conclusion & thanks

- Many exciting developments in computational electromagnetics
- All examples from this talk solved with the open source codes Gmsh and GetDP — more examples on http://onelab.info
- Superconducting wire example developed in collaboration with
  - Antti Stenvall (Tampere University)
  - Abelin Kameni (Supelec & Université Paris XI)

  Available on http://onelab.info/wiki/Superconducting_wire
- For more information (references, preprints, codes, etc.), see http://www.montefiore.ulg.ac.be/~geuzaine

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