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Invited presentation 5MOr1A-01 given at ASC 2016; Denver,Colorado,USA, September 4 – 9, 2016.

UNITED STATES DEPARTMENT OF COMMERCE

National Institute of Standards and Technology

Extrapolative Scaling Expression:

A fitting equation for extrapolating full $I_c(B,T,\varepsilon)$ data matrixes from limited data

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Organization of Talk

1. Context for Extrapolative Scaling Expression (ESE, or "easy")

   Result of three SUST invited topical reviews:

   Part 1 – Organization of many parameterizations of USL into separable parts

   Part 2 – Derivation of ESE from raw scaling data (> 4000 I_c measurements)

   Part 3 – Applications

2. Focus on new extrapolation capabilities made possible with ESE

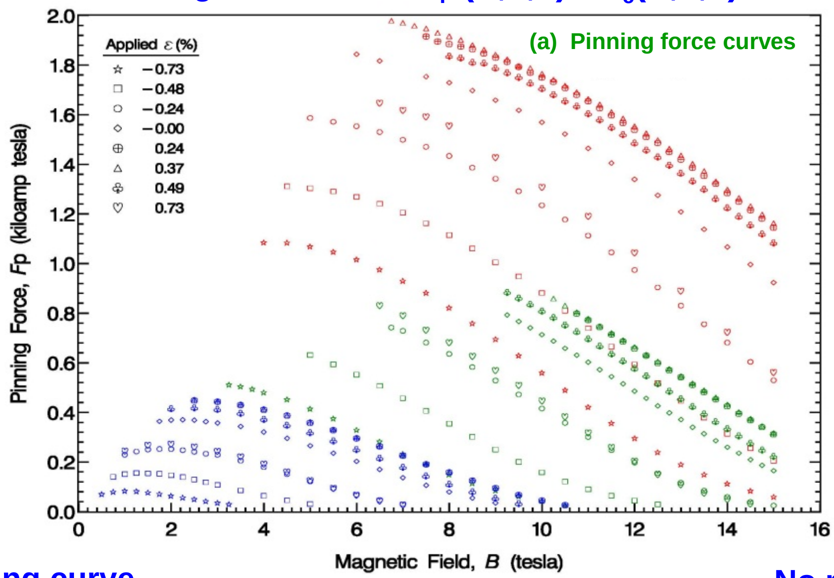
   Emphasis on *concatenation* of errors (not included in SUST articles)

   Illustrate with practical conductors: HL-LHC, ITER, NMR cryo-cooled magnets

3. Suggestions for future research

Two approaches in use to analyze $I_c(B,T,\varepsilon)$

Pinning force curves $F_P(B,T,\varepsilon) = I_c(B,T,\varepsilon)\,B$



Fundamental Scaling

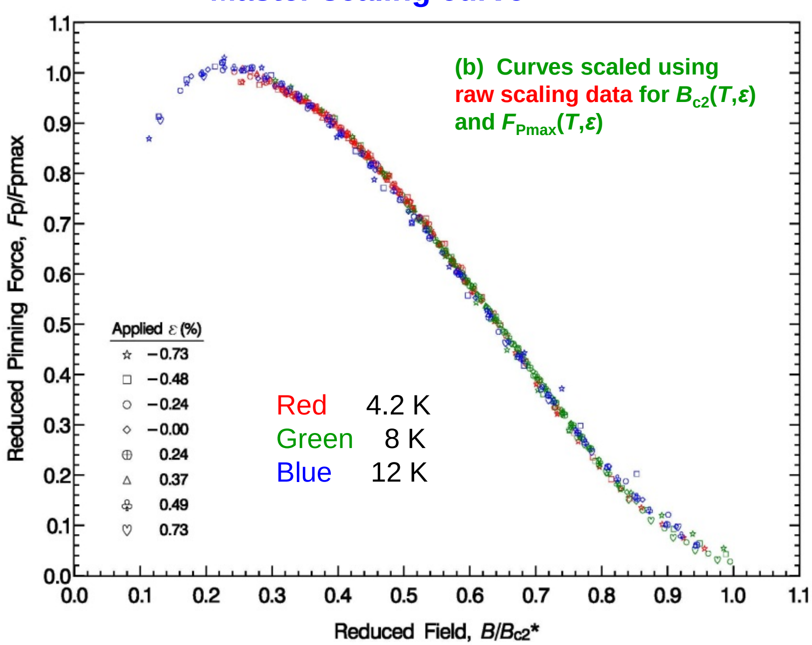
Curve registration

1) $F_{Pmax}(T,\varepsilon)$
2) $B_{c2}^*(T,\varepsilon)$

All we get

Extrapolation capability

Master scaling curve

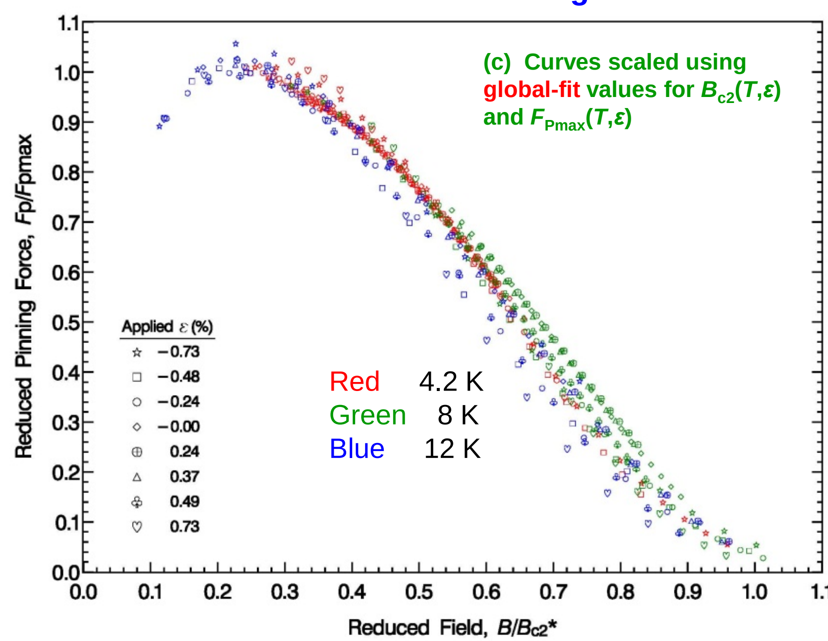


Global Fitting Equations

$$I_c(B,T,\varepsilon)\,B = C\,[b_{c2}(\varepsilon_0)]^s\ x$$
$$x\ (1 - t^{\nu})^{\eta-\mu}\,(1 - t^2)^{\mu}\ b^p(1-b)^q$$

Interpolation only

No master scaling curve



ESE is a fitting equation for the 3-dimensional $I_c(B,T,\varepsilon)$, which is:

1. Derived from an extensive one-time analysis of raw scaling data.
2. But simply applied as a fitting equation (without analyzing raw scaling data).
3. And, unlike present fitting equations, it has the *extrapolation capability of* fundamental scaling. (Reason? – based on master scaling curves; it is not empirical or semi-empirical)

Because no theoretical assumptions were made in its derivation, the results also serve to evaluate underlying semi-theoretical models for general par. of USL

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Derivation of the Extrapolative Scaling Expression (ESE)

True Scaling

Registration gives:

1) $K(T,\boldsymbol{\varepsilon})$
2) $B_{c2}^{*}(T,\boldsymbol{\varepsilon})$

} Raw data → Scaling Constants / Fitting Parameters }

- Extrapolation capability

ESE Fitting Equation

$$I_c(B,T,\boldsymbol{\varepsilon})\,B = C\,b_{c2}(\boldsymbol{\varepsilon})^{s}\,(1-t^{1.5})^{\eta-\mu}(1-t^{2})^{\mu}\,b^{p}(1-b)^{q}$$

reduced variables: $b \equiv B/B_{c2}^{*}(\boldsymbol{T},\boldsymbol{\varepsilon})$ and $t \equiv T/T_{c2}^{*}(\boldsymbol{\varepsilon})$

where: $B_{c2}^{*}(T,\boldsymbol{\varepsilon})/B_{c2}^{*}(0,0) = (1-t^{1.5})\,b_{c2}(\boldsymbol{\varepsilon})$

$$T_c^{*}(\boldsymbol{\varepsilon}) = T_c^{*}(0)\,b_{c2}(\boldsymbol{\varepsilon})^{1/3}$$

- Extrapolation capability
- But, in an easy way

Three scaling constants*:*

$w = 3.0 \pm 0.03$
$v = 1.5 \pm 0.04$
$u = 1.7 \pm 0.1$

Stable with respect to:

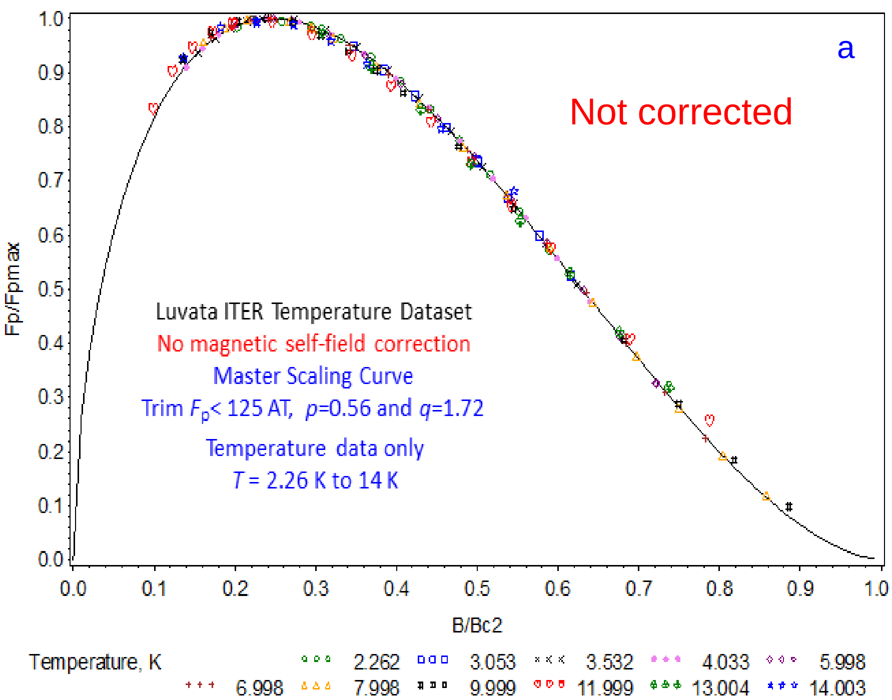
- conductor type
- trim factors
- p and q values
- magnetic self-field correction

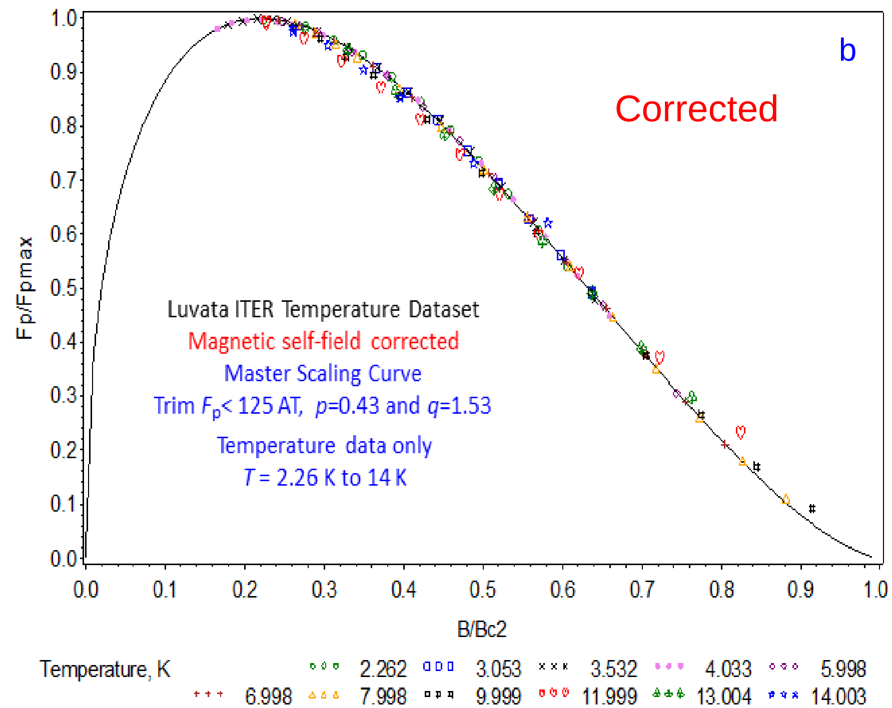
Magnetic Self-field Correction

Needed for comparisons -- short-sample data, different apparatus, magnetization

Large effect on flux-pinning curve, BUT:

1. F_P curves still scale into master curve
2. Scaling constants *w, v,* and *u* unchanged by SF correction





Bottom line: raw scaling analysis gives:

Extrapolative Scaling Expression (ESE), the "easy" fit.

Most useful form:

$$I_c(B,T,\varepsilon)\,B = C\,b_{c2}(\varepsilon)^{s}\,(1-t^{1.5})^{\eta-1}(1-t^{2})\,b^{p}(1-b)^{q}$$

where $b \equiv B/B_{c2}^{*}(T,\varepsilon)$ is the reduced field, and $t \equiv T/T_c^{*}(\varepsilon)$ is the reduced temperature

$$B_{c2}^{*}(T,\varepsilon)/B_{c2}^{*}(0,0) = (1-t^{1.5})\,b_{c2}(\varepsilon)$$

$$T_c^{*}(\varepsilon) = T_c^{*}(0)\,b_{c2}(\varepsilon)^{1/3}$$

and fitting parameters C & $B_{c2}^{*}(0,0)$, and 4 core parameters $T_c^{*}(0)$, s, η, & C_1 (in $b_{c2}(\varepsilon)$).

Hybrid temperature models with η fitted (Durham) and $\mu = 1$ (Twente) have the following advantages:

- Overall fitting accuracy
- Parameter consistency (η variability < half that of μ)
- ***Extrapolation*** capability to temperatures *below 4 K* (~1 % errors)

Exponential strain model for $b_{c2}(\varepsilon)$:

- One fitting parameter C_1 (*strain sensitivity index, default values*)
- 3-D strain capability
- ***Extrapolation*** capability to high compressive strains

Applications of the Extrapolative Scaling Expression (ESE)

N.B. -- Fitting F_P, not I_c . Errors consistently one-fifth !

Extrapolation capability in four new areas:

→ 1. Five-fold reduction in measurement space: extrapolate minimum dataset
(reduces weeks for full $I_c(B\text{-}T\text{-}\varepsilon)$ measurements to a few days)

2. Combination of data from separate T and ε apparatuses
(offers flexibility and productive use of limited data)

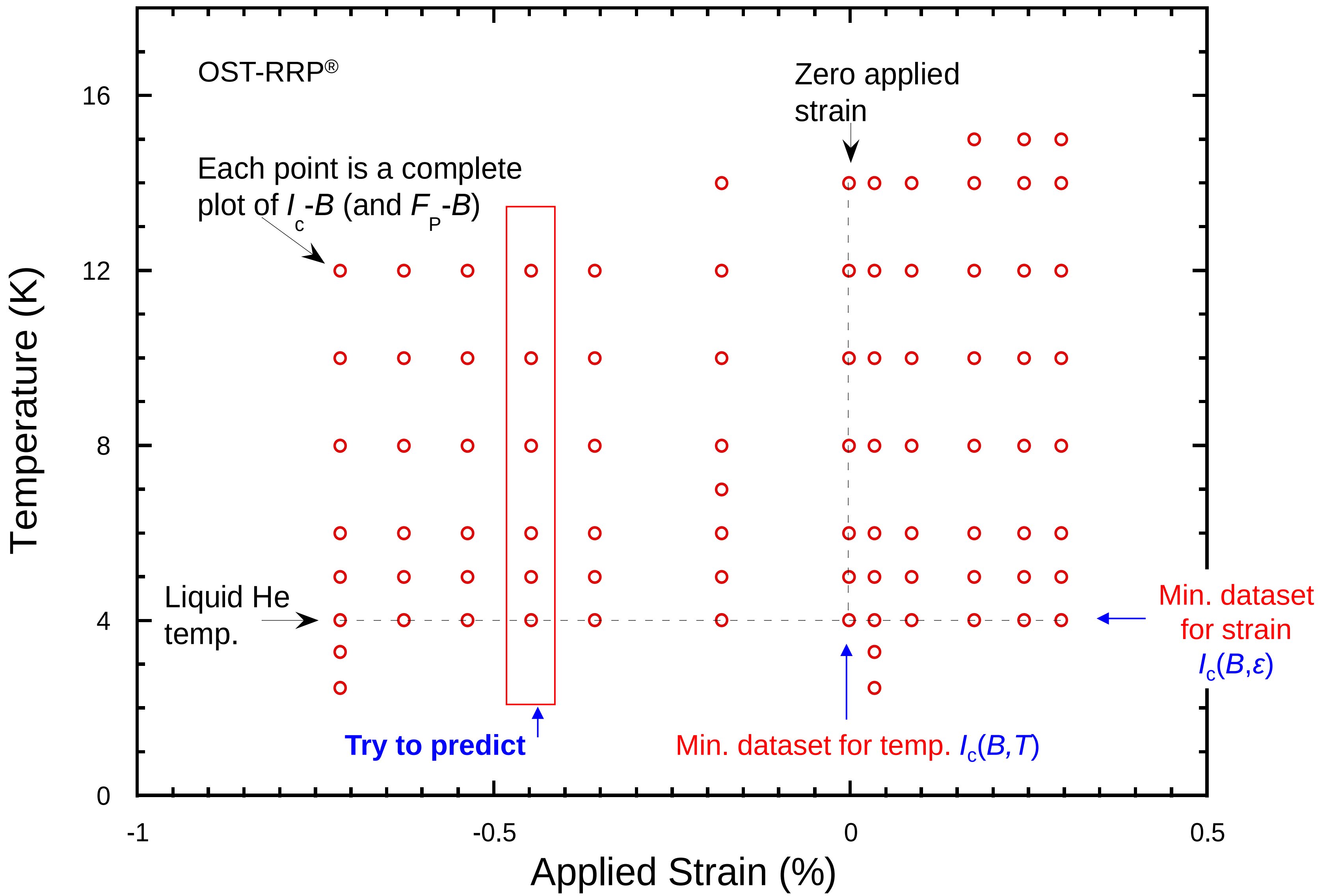
3. Full $I_c(B,T,\varepsilon)$ extrapolation from as little as a single $I_c(B)$ curve
(useful for production sample measurements, e.g., HL-LHC, FCC)

4. Interpolation with option for nearby extrapolations
(with *default core* parameters)

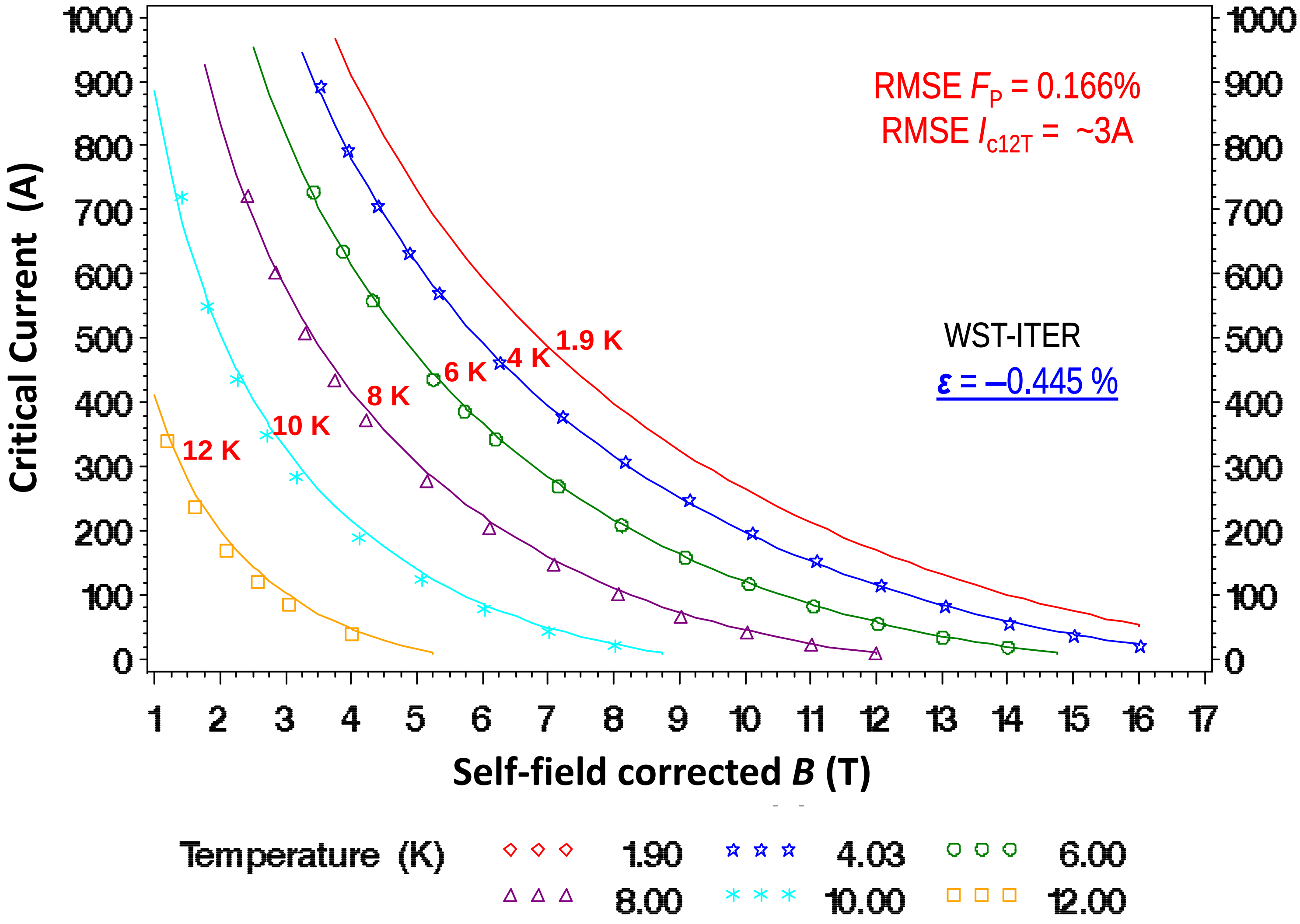
Minimum Dataset for extrapolating full $I_c(B,T,\varepsilon)$ characteristics – derived from scaling

Visualize with *T-ε measurement map*

5-fold reduction in measurement space



Minimum dataset extrapolation with ESE-Hybrid model



Applications of the Extrapolative Scaling Expression (ESE)

Extrapolation capability in three new areas:

1. Five-fold reduction in measurement space for unified B-T-ε apparatuses

   (reduces weeks for full $I_c(B\text{-}T\text{-}\varepsilon)$ measurements to a few days)

2. → Combination of data from separate T and ε apparatuses

   (offers flexibility and productive use of limited data)

3. Full $I_c(B,T,\varepsilon)$ extrapolation from as little as a single $I_c(B)$ curve

   (useful for production measurements, e.g., HL-LHC, FCC)

4. Interpolation with option for nearby extrapolations

   with default *core* parameters

Combining limited datasets (examples in Part 3)

Core parameters – depend only on ratios of raw scaling data. Very stable.

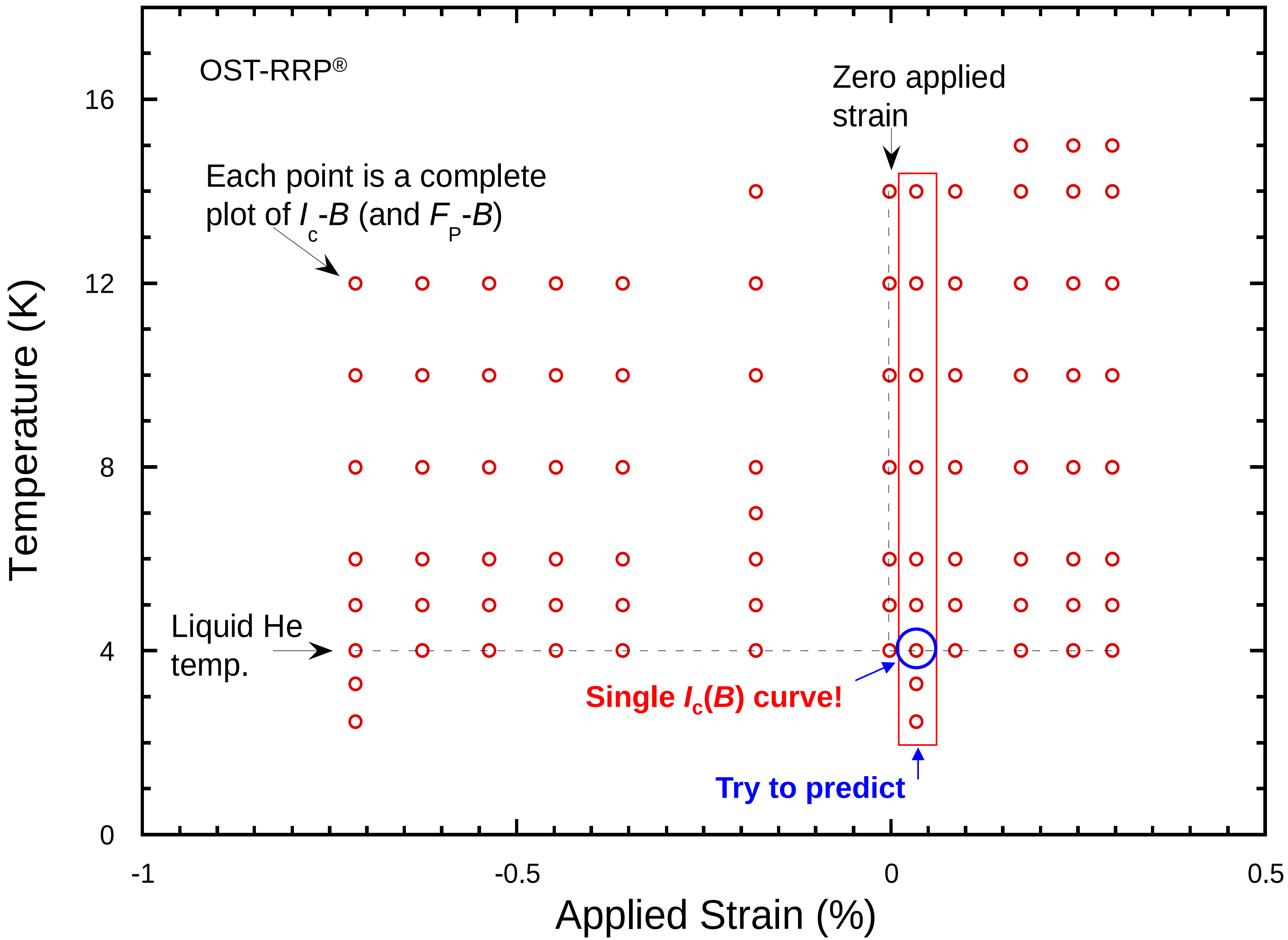
Transfer among similar conductors (same comp., config, and heat treatment)

Available data	Parameters determined							
			Core Parameters					
	C	$B_{c2}^*(0,0)$	$T_c^*(0)$	η	s	$b_{c2}(\varepsilon)$	p	q
$I_c(B,T,\varepsilon)$ (unified T,ε apparatus)	✓	✓	✓	✓	✓	✓	✓	✓
$I_c(B,T)$ fixed ε (dedicated T rig)	✓	✓	✓	✓			✓	✓
$I_c(B,\varepsilon)$ fixed T (dedicated ε rig)	✓	✓			✓	✓	(✓)	✓
$I_c(T)$ fixed B,ε (dedicated T rig)	✓		✓	✓				
$I_c(\varepsilon)$ fixed B,T (dedicated ε rig)	✓					✓		
$I_c(B)$ fixed T,ε (routine I_c testing)	✓	✓					(✓)	✓
I_c fixed B,T,ε (routine I_c testing)	✓							

Min. dataset: $I_c(B,T)$ fixed ε and $I_c(B,\varepsilon)$ fixed T

Single $I_c(B)$ curve: $I_c(B)$ fixed T,ε

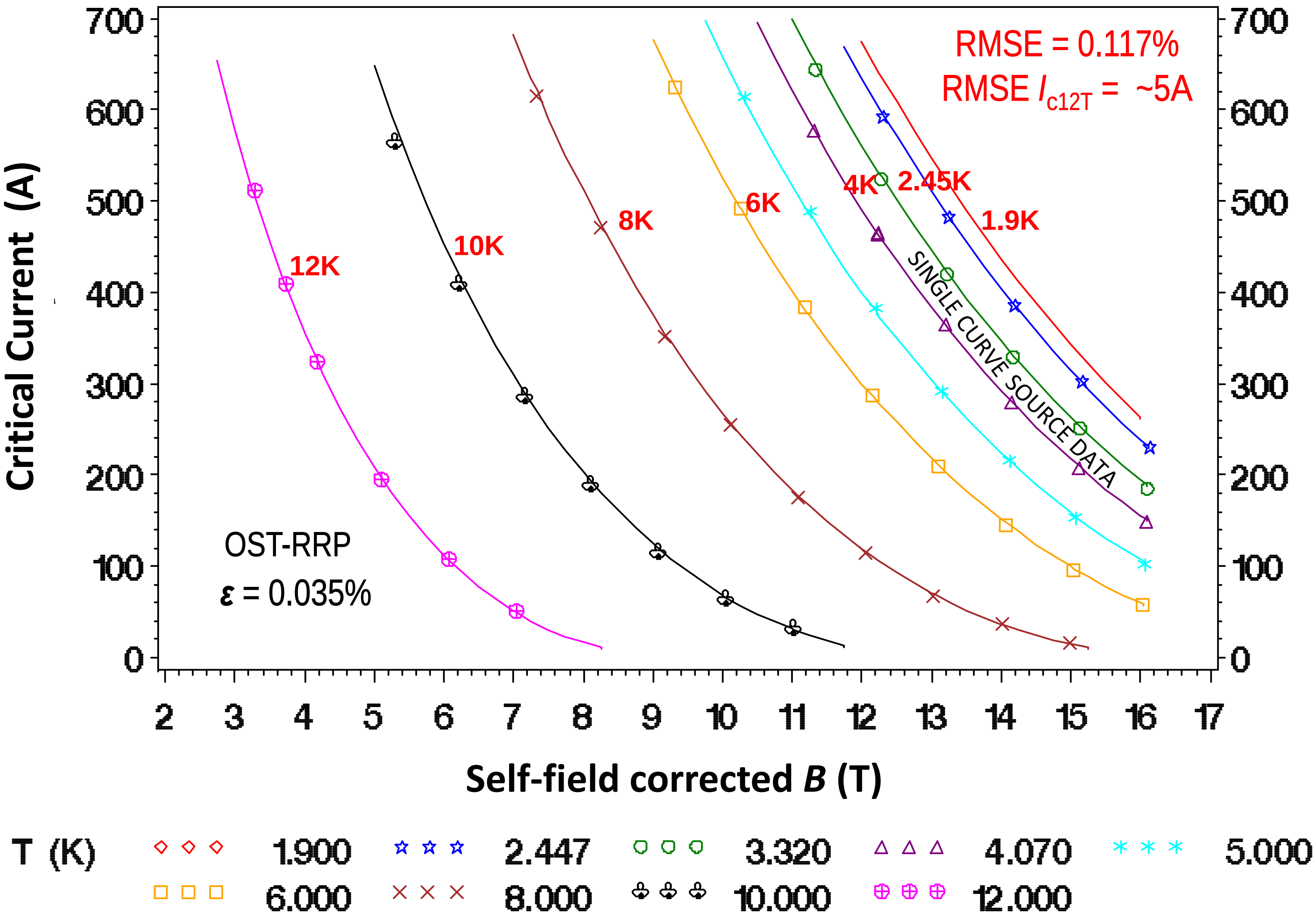
Single $I_c(B)$ curve extrapolation



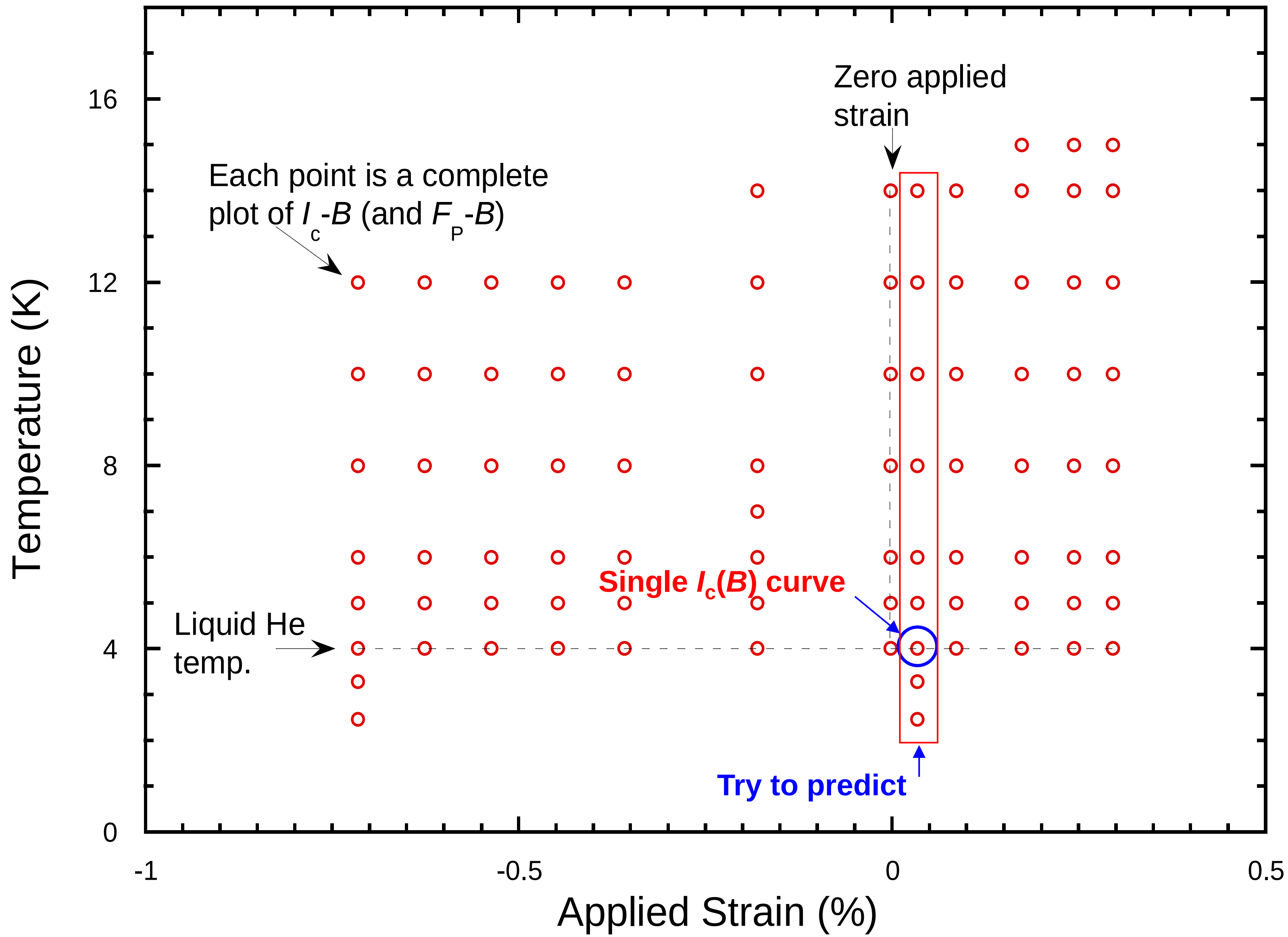
Single $I_c(B)$ curve extrapolation

From point in T-ε map at 4.07 K and 0.035 % strain

Core parameters from minimum dataset

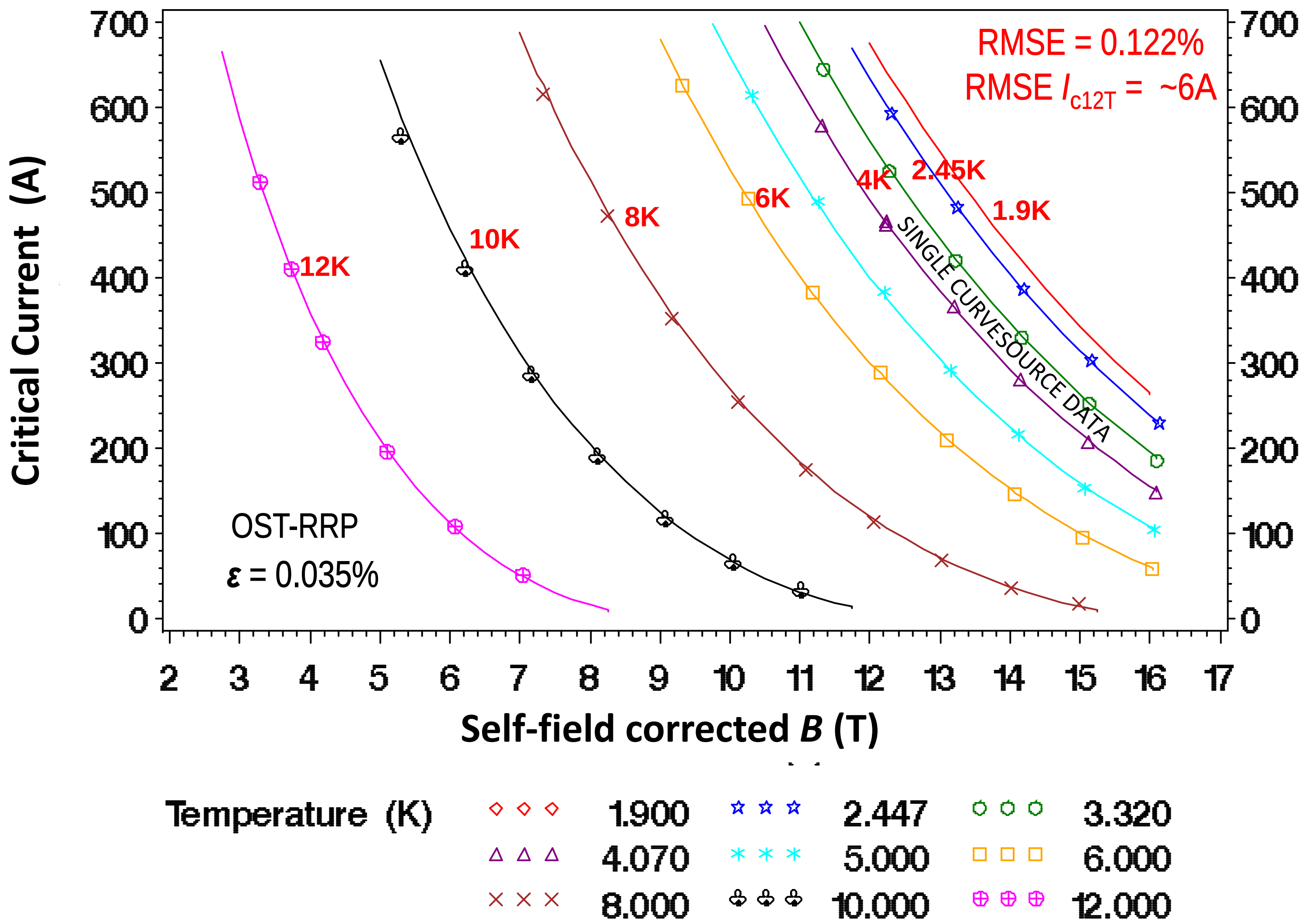


Single $I_c(B)$ curve extrapolation
Core parameters from min. dataset with data only > 4 K

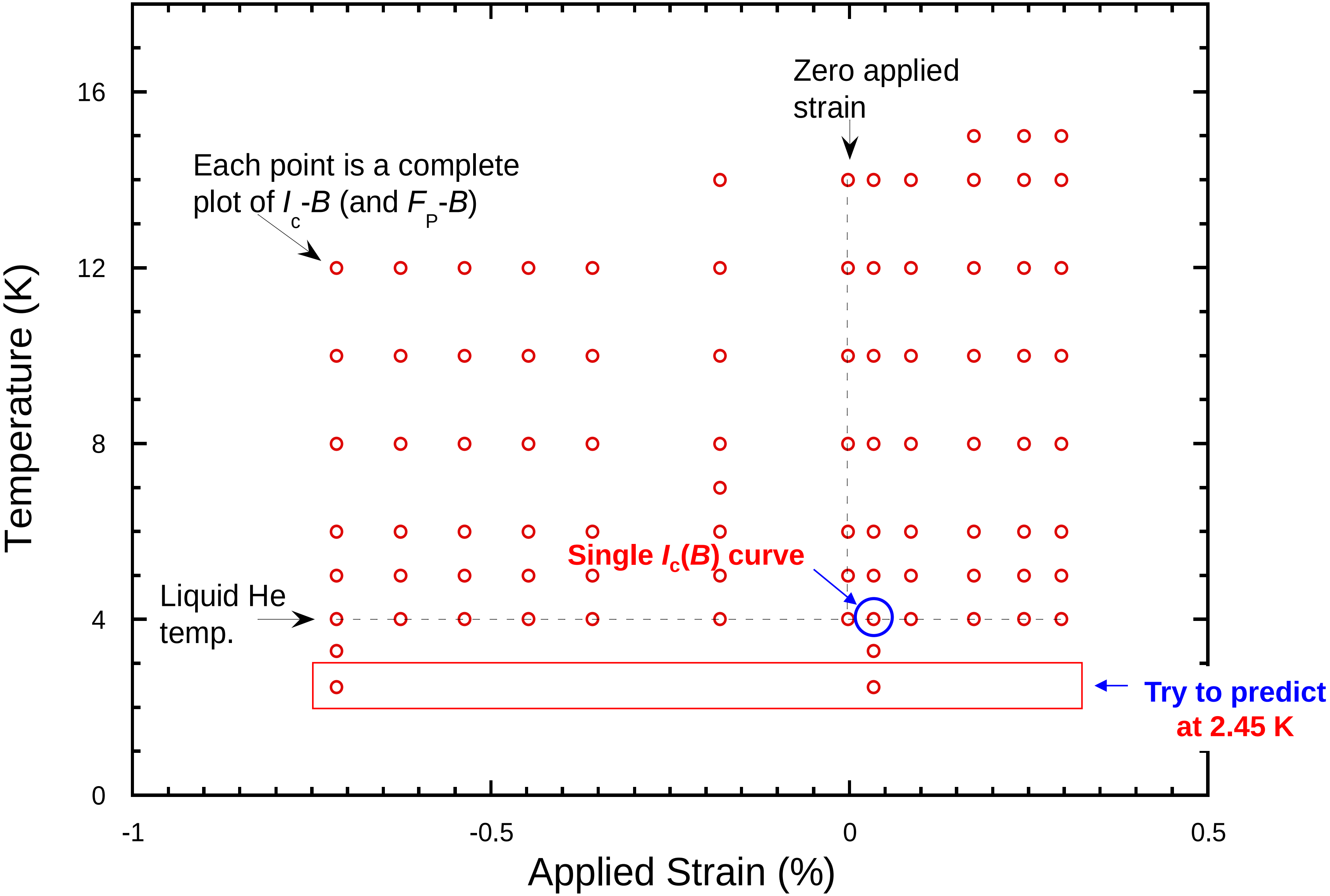


Single $I_c(B)$ curve extrapolation from 4.07 K and 0.035 % strain

Core par. from minimum dataset, using data only > 4 K (combine 3 extrapolations)

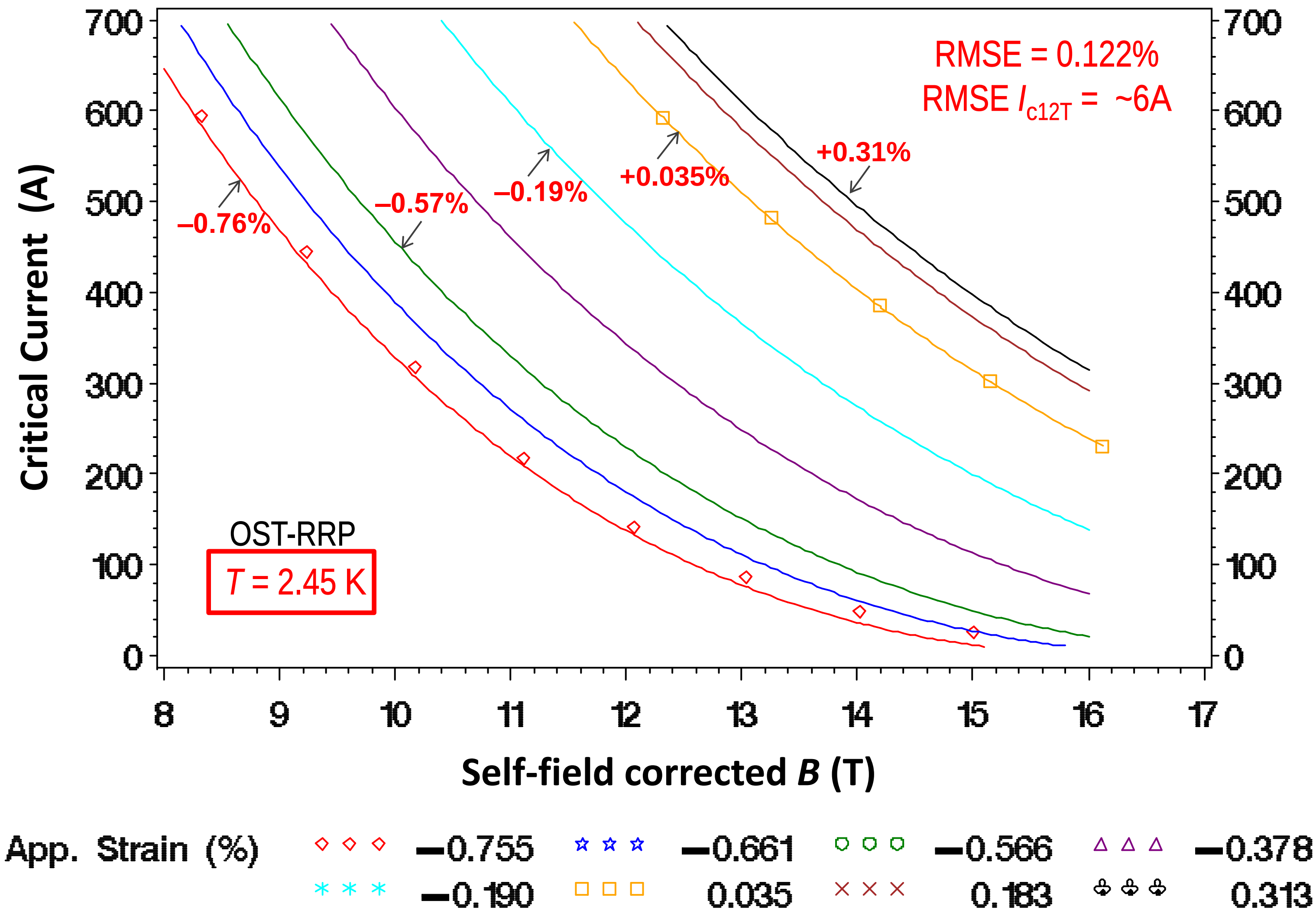


Single $I_c(B)$ curve strain extrapolation
Core par. from minimum dataset, using data only > 4 K



Single $I_c(B)$ curve strain extrapolation from 4.07 K and 0.035 % strain

Core parameters from minimum dataset, using data only > 4 K



Caveats:

1. Evaluated intrinsic errors for predicting the ***non-core*** parameters from a single $I_c(B)$ curve

2. Extrinsic errors need to be minimized. ***Core*** parameters determined from:
   - Samples with similar configuration, doping, and heat treatment (e.g., production samples).
   - Similar sample holders (minimize strain variability)
     Matching material preferred (thermal contraction strain)
     Continuously soldered preferred to provide good F_L support
     Cu-Be holders easy solution (avoids unsupported conductor settling)

→ If control extrinsic errors, such extrapolations quite effective for similar conductors.

Applications of the Extrapolative Scaling Expression (ESE)

Extrapolation capability in three new areas:

1. Five-fold reduction in measurement space for unified B-T-ε apparatuses
   (reduces weeks for full $I_c(B$-T-$\varepsilon)$ measurements to a few days)
2. Combination of data from separate T and ε apparatuses
   (offers flexibility and productive use of limited data)
3. Full $I_c(B,T,\varepsilon)$ extrapolation from as little as a single $I_c(B)$ curve
   (useful for production measurements, e.g., HL-LHC, FCC)
4. → Interpolation with option for nearby extrapolations
   with default *core* parameters when data limited

Table A1.1. The ESE parameter set, with Hybrid $h(t)$ and the *Exponential* parameterization of $b_{c2}(\varepsilon)$ for data *not corrected for magnetic self field.*

			Core Scaling Parameters								
Nb_3Sn Conductor	C (AT)	$B_{c2}^*(0,0)$ (T)	$T_c^*(0)$ (K)	η	s	ε_{I0}**‡ (%)	C_1‡	p†	q†	RMSFD (%)	RMS (%)
OST-RRP®	50,514	29.09	16.94	2.254	1.150	−0.355	0.748	0.5	2.061	9.0	0.120
WST-ITER	21,015	31.02	16.81	2.025	1.388	−0.302	0.817	0.573	1.834	4.8	0.114
LUVATA	14,955	29.70	16.43	1.966	1.4	−0.321	0.657	0.562	1.703	2.0	0.078
VAC	7,631	29.91	16.84	2.002	1.097	−0.313	0.923	0.480	1.445	4.6	0.247
EMLMI	11,920	30.79	17.02	2.380	0.874	−0.271	1.139	0.5	1.835	3.6	0.170

Table A1.2. The ESE parameter set, with Hybrid $h(t)$ and the *Invariant* parameterization of $b_{c2}(\varepsilon)$ for data *not corrected for magnetic self field.*

			Core Scaling Parameters										
Nb_3Sn Conductor	C (AT)	$B_{c2}^*(0,0)$ (T)	$T_c^*(0)$ (K)	η	s	ε_m** (%)	c_2	c_3	c_4	p†	q†	RMSFD (%)	RMS (%)
OST-RRP®	47,954	27.58	16.65	2.252	1.210	0.302	1.016	0.717	0.183	0.5	2.061	7.3	0.104
WST-ITER	19,772	29.62	16.53	2.023	1.356	0.305	0.823	0.424	0.118	0.577	1.855	4.5	0.106
LUVATA	14,166	28.60	16.21	1.966	1.4	0.323	0.660	0.669	1.136	0.562	1.709	2.0	0.082
VAC	7,654	28.92	16.45	1.972	1.040	0.311	0.893	0.376	0.053	0.512	1.549	4.5	0.219
EMLMI	11,419	28.81	16.71	2.405	0.851	0.273	1.051	0.610	0.258	0.5	1.883	3.6	0.156

Default Core Parameters

Survey of core values for fully optimized ternary high-J_c Nb_3Sn → average default values:

$T_c^*(0)$ = 16.7 K
η = 2.0 (ITER) – 2.2 (RRP)
s = 1.2 (RRP) – 1.4 (ITER)
p = 0.5 and q = 2.0.

Additional meas. → "catalog" by generic conductor category
(e.g., Ti vs. Ta doping, RRP, internal Sn, etc.)

Future Work

Immediate need: (huge dividends)

1. * Measure $I_c(B,T)$ above 4.2 K for at least one conductor of the RRP and PIT production wires for the Hi-Lumi (to obtain T_c* and η).

Longer term:

2. Compile core parameters in different types of Nb_3Sn – catalog values
3. Evaluate accuracy of ESE in extreme regions of B-T-ε space for magnet modeling
4. Magnetization vs. transport I_c data
5. Assess if scaling constants hold for artificial-pinning-center architectures
6. ESE relationship for BSCCO, MgB_2, Nb_3Al, YBCO? (master curve → extrapolation)

Conclusion

- ESE is based on fundamental raw scaling data
- But unlike fundamental scaling, applied as a fitting equation– quick, straightforward
- Simple, robust, and
- Can interpolate and extrapolate with excellent accuracy → significant time savings

Excel source data & ESE spreadsheet tool at www.ResearchMeasurements.com
SUST invited topical review articles