Power dissipated by trapped vortices under a strong RF field and Campbell penetration depth in superconducting resonant cavities

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Flux vortices and transport currents in type II superconductors*

A. M. CAMPBELL and J. E. EVETTS Department of Metallurgy and Materials Science, University of Cambridge, Cambridge, UK

Abstract

This article is concerned with the mechanisms by which type II superconductors can carry currents. The equilibrium properties of the vortex lattice are described and the generalized driving force in gradients of temperature and field is derived using irreversible thermodynamics. This leads to expressions for thermal cross effects which can include pinning forces.



Archie Campbell

Alex Gurevich

Department of Physics, Old Dominion University, Norfolk, VA, USA

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Campbell's penetration depth: good work always stays young and popular

J. PHYS. C (SOLID ST. PHYS.), 1969, SER. 2, VOL. 2. PRINTED IN GREAT BRITAIN

The response of pinned flux vortices to low-frequency fields

A. M. CAMPBELL Department of Metallurgy, University of Cambridge MS. received 21st April 1969 J. Phys. C: Solid St. Phys., 1971, Vol. 4. Printed in Great Britain

The interaction distance between flux lines and pinning centres

> A. M. CAMPBELL Department of Metallurgy and Materials Science, University of Cambridge, UK MS. received 22nd July 1971

RAPID COMMUNICATIONS

PHYSICAL REVIEW B 84, 060509(R) (2011)

PHYSICAL REVIEW B 93, 064515 (2016) Probing the pinning landscape in type-II superconductors via Campbell penetration depth

Magnetic-field-dependent pinning potential in LiFeAs superconductor from its Campbell penetration depth

Plengchart Prommapan,^{1,2} Makariy A. Tanatar,¹ Bumsung Lee,³ Seunghyun Khim,³ Kee Hoon Kim,³ and Ruslan Prozorov^{1,2,+} ¹The Ames Laboratory, Ames, Iowa 50011, USA R. Willa, V. B. Geshkenbein, and G. Blatter Institute for Theoretical Physics, ETH Zurich, 8093 Zurich, Switzerland

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PHYSICAL REVIEW B 67, 184501 (2003)

Campbell penetration depth of a superconductor in the critical state

R. Prozorov,^{1,2} R. W. Giannetta,¹ N. Kameda,³ T. Tamegai,³ J. A. Schlueter,⁴ and P. Fournier⁵
¹Loomis Laboratory of Physics, University of Illinois at Urbana–Champaign, 1110 West Green Street, Urbana, Illinois 61801

R. Willa, V. B. Geshkenbein, and G. Blatter Institute for Theoretical Physics, ETH Zurich, 8093 Zurich, Switzerland

PHYSICAL REVIEW B 92, 134501 (2015)

Campbell penetration in the critical state of type-II superconductors

PRL 115, 207001 (2015)

PHYSICAL REVIEW LETTERS

week ending 13 NOVEMBER 2015

Campbell Response in Type-II Superconductors under Strong Pinning Conditions

R. Willa,¹ V. B. Geshkenbein,¹ R. Prozorov,² and G. Blatter¹ ¹Institute for Theoretical Physics, ETH Zurich, 8093 Zurich, Switzerland ²The Ames Laboratory and Department of Physics and Astronomy, Iowa State University, Ames, Iowa 50011, USA (Baselund 16 July 2015, sublided 11 Neurophyr 2015) Coexistence of long-range magnetic order and superconductivity from Campbell penetration depth measurements

OPERCONDUCTOR SCIENCE AND TECHNOLOGY

444 00.1088/0953-2048/22/3/03400

R Prozorov, M D Vannette, R T Gordon, C Martin, S L Bud'ko and P C Canfield

Ames Laboratory, Iowa State University, Ames, IA 50011, USA

Supercond. Sci. Technol. 22 (2009) 034008 (7pp)

Department of Physics and Astronomy, Iowa State University, Ames, IA 50011, USA

Superconducting linac applications

Spallation neutron source (ORNL)



X-ray free electron laser





"dog bone" damping ring

Superconducting RF cavities resonating at 0.1-2 GHz Currently made of pure niobium Cooled by superfluid helium at 2K Tens of thousands of these in miles long tunnels



Vacuum Insulation



Definitions

- Δ superconducting gap
- ξ coherence length
- λ magnetic penetration depth
- ρ_n normal state resistivity
- R_s surface resistance
- Q quality factor

 $\omega = 2\pi f - \text{RF}$ circular frequency $\eta_0 = \phi_0^2 / 2\pi \xi^2 \rho_n$ - Bardeen-Stephen vortex drag coefficient

 ϕ_0 - magnetic flux quantum

 ℓ - spacing between a pinning center and the surface

 v_0 - Larkin-Ovchinnikov (LO) critical velocity of a vortex

- κ thermal conductivity
- d thickness of a cavity wall

 α_{K} - Kapitza thermal conductance between a cavity wall and liquid He

 $\epsilon = g \phi_0^2 / 4 \pi \mu_0 \lambda^2$ - vortex line tension

 $g = \ln(\lambda/\xi) + 1/2$

- T_c critical temperature
- B_{c1} lower critical field
- B_c thermodynamic critical field
- B_s superheating field
- B_{c2} upper critical field

Quality factor

$$Q = \frac{\omega\mu_0 \int_V |\mathbf{H}(\mathbf{r})|^2 dV}{\oint_A R_s |\mathbf{H}(\mathbf{r})|^2 dA} = \frac{G}{\langle R_s \rangle},$$

$$G = \alpha G_0, \qquad \qquad G_0 = \mu_0 c = 377 \,\Omega$$

Surface resistance of good normal metals

$$R_s = (\pi \mu_0 f \rho_n)^{1/2}$$
$$Q \sim 10^5 - 10^6$$

Mean EM energy Mean dissipated power Vacuum impedance

Clean Cu with ρ_n = 10⁻¹⁰ Ω m at f = 0.5-2 GHz has R_s = 0.5-1 m Ω

Exponentially-small BCS surface resistance of superconductors:

$$R_s \simeq \frac{\mu_0^2 \omega^2 \lambda^3}{\rho_n kT} \ln\left(\frac{9kT}{2\hbar\omega}\right) \exp\left(-\frac{\Delta}{kT}\right) \simeq 2 - 10 \, n\Omega, \quad @1.7 - 2K, \ 1 - 2GHz$$

 $Q \sim 10^{10} - 10^{11}$

How good can Nb cavities be?



A. Grassellino and S. Aderhold, TTC meeting, Saclay, France (2016)

Continuous progress in improving Q(H) and E_{acc} in Nb cavities.

Understanding the fundamental limits of Q(H) and the SRF accelerating gradients

The RF field of H = 200 mT induces current densities at the surface close to the BCS pairbeaking limit.

High Q can only be achieved in the Meissner state with a small density of trapped vortices.

Why are trapped vortices so bad for SRF cavities?



- Vortices get trapped by materials defects on cooling the cavity through T_c at which H_{c1}(T) vanishes.
- Trapped vortices caused by Earth's magnetic field can produce ~ 10² higher RF losses than the BCS surface resistance at 2K and 1-2 GHz.



London penetration depth of $\approx 40 \text{ nm} \ll d = 3 \text{mm}$

Even good screening (1% of H_E) cannot eliminate trapped vortices. Temperature maps have revealed sparse hotspots of vortex bundles which reduce the quality factors and breakdown fields: Vogt, Kugeler and Knobloch, PRAB 18, 042001 (2015);
 Gonnella, Kaufman and Liepe, JAP 119, 073904 (2016); Dhakal et al, PRAB 23, 023102 (2020).

Detection and manipulation of trapped vortices

In films vortices are observed using scanning SQUID, Kirtley, Rep. Prog. Phys. 73, 126501 (2010) MO imaging, STM, MF, Lorentz microscopy, ... hotspots

Arrays of carbon sensors to get local temperature maps with the sensitivity of a few mK and spatial resolution of a few mm (Cornell, Jlab, FNAL)





Flushing vortices out by strong thermal gradients or scanning laser beams: Ciovati and Gurevich, PRAB 11, 122001 (2008); Gurevich and Ciovati. PRB 87, 054502; (2013); Romanenko et al, JAP 115, 184903 (2014); Posen et al, JAP 119, 213903 (2016).

Key issues

- Trapped vortices can produce significant losses which can be much higher than the BCS losses in SRF resonator cavities.
- Vortex losses are determined by an effective Campbell penetration depth
- New physics of superfast vortices driven by strong Meissner screening currents at the depairing limit in SRF cavities.
- How fast can vortices move? How long does it take for a vortex to penetrate a superconductor?
- Nonlinear dynamics of supersonic vortices: field-dependent RF losses, Larkin-Ovchinnikov instability, decrease of the surface resistance with the RF amplitude, ...
- How much vortex dissipation can be tolerated? Can vortex dissipation be mitigated by strong pinning?

Trapped vortex driven by RF Meissner current

An elastic vortex is driven by the Lorentz force ${f f}_L=\phi_0{f J} imes{f z}$ perpendicular to J: $J(z,t)=(H_a/\lambda)e^{-z/\lambda}\sin\omega t$



The surface Lorentz force is balanced by viscous drag force and bending stress

At $H_a = 100-200 \text{ mT}$, J(0) approaches the depairing limit

$$J_d \simeq H_c/\lambda$$

Typical depinning $J_c = 10-100 \text{ kA/cm}^2$ in Nb are some 4 orders of magnitude lower than $J_d = H_c/\lambda$, = 500 MA/cm²

Pinning is too weak to stop the vortex tip at the surface above $H > 0.01H_c = 2 \text{ mT}$

RF Campbell length



- Campbell length L_ω can be much greater than λ.
- L_ω can be either larger or smaller than the pin distance from the surface.
 If ℓ > L_ω the effect of pinning is weak

Dynamic eq for displacements u(x,t) of a vortex driven by a weak RF field H_a << H_c

$$\eta \dot{u} = \epsilon u'' - (H_a/\lambda)e^{-x/\lambda}\sin\omega t$$

Elastic RF ripple length – Campbell penetration depth:

$$L_{\omega} = \sqrt{rac{\epsilon}{\eta \omega}} = rac{\xi}{2\lambda} \sqrt{rac{g
ho_n}{\pi \mu_0 f}}$$

Clean Nb

$$\begin{split} \lambda &\approx \xi, \quad \rho_n = 1 \ n\Omega m, \quad f = 2 \ GHz \\ L_\omega &\approx 180 \ nm \end{split}$$

Nb₃Sn

$$\begin{split} \lambda/\xi \approx 20, \quad \rho_n = 0.2 \ \mu\Omega m, \quad f = 2 \ GHz \\ L_\omega \approx 126 \ nm \end{split}$$

Low-field RF power of an oscillating vortex



• High ω : $\frac{L_{\omega} \leq \lambda}{P_{\infty}} = \pi H^2 \rho_n \xi^2 / 2\lambda$

No dependence on the pin spacing

 $P\sim 0.13~\mu W$ at B = 100 mT and 2 GHz.

Hotspots revealed by thermal maps require regions \sim few mm with $\sim 10^6$ vortices

Extreme dynamics of vortex tips at the surface

At H = H_c, the superflow velocity of Cooper pairs reaches the critical pairbreaking value $v_c = \Delta/p_F$.

How fast can the vortex tip move at the pairbreaking limit?

$$v \simeq \frac{J_d \phi_0}{\eta} \simeq \frac{\rho_n \xi}{2\mu_0 \lambda^2}$$

This rough estimate yields v = 10 km/s, which exceeds both the speed of sound (2-4 km/s) and $v_c = \Delta/p_F = 1 \text{ km/s}$

How can a supersonic vortex tip remain connected to a subsonic elastic vortex line in the bulk?

SRF cavities are a unique testbed to study the extreme dynamics of a vortex driven by nondissipative Meissner currents at the pairbreaking limit



How can a vortex move faster than the current superflow which propels it?



- Vortex core stretches along the direction of motion
- Vortex can move much faster than the drift velocity of supercurrent
- $\circ~$ V can exceed the pairbreaking velocity



A sailboat can move much faster than the wind if drag is weak and the sail is nearly perpendicular to the wind blow.

What does experiment say?



75 nm thick Pb film: imaging of penetrating vortices with a nanoscale SQUID on tip

Velocities can reach 10-20 km/s as J(x,y) at the edge reaches J_d (H = H_s for the SRF cavities)

If v = 10 km/s, a vortex penetrates by the distance

$L \simeq v/f \simeq 10 \mu m \gg \lambda$, @ 1GHz

Vortices penetrate almost instantaneously through the Meissner RF layer

Hot vortex branching trees. No materials defects can stop such superfast vortices.



Dynamics of vortex branching observed by SOT microscope

Pb bridge at $B_a = 27 \text{ G}$ SOT diameter: 225 nm Scan area: $12 \times 12 \mu m^2$ Pixel size: 40nm Scan time: 4 min/frame T = 4.2 K



L. Embon et al, Nature Comm. 8, 85 (2017)

What happens to the vortex core at high velocities?



- A nearly round vortex core of radius $\approx \xi$
- Cloud of dissipative quasiparticles is locked onto the moving core



 $\eta(v) \simeq$

Vortex drag decreases with v:

Velocity dependence of $\eta(v)$

Larkin-Ovchinnikov mechanism

Reduction of the vortex drag due to diffusive depletion of quasiparticles in the moving core

 $\eta(v)\simeq rac{\eta_0}{1+(v/v_0)^2}$

LO critical velocity:

$$v_0 \sim (D/\tau_E)^{1/2} (1 - T/T_c)^{1/4}$$

D is the electron diffusivity

The energy relaxation time $\tau_E(T)$ caused by inelastic e-p scattering increases as T decreases so v₀(T) is expected to decrease at T << T_c

Electron overheating

The drag coefficient depends on the electron temperature T_v of the vortex

$$\eta(T_0) = \frac{\phi_0 B_{c2}(0)}{\rho_n} \left(1 - \frac{T_v}{T_c}\right)$$

2D power balance:

$$\eta(T_v)v^2 \simeq (T_v - T_0)\kappa$$

An effective thermal conductivity $\kappa(T_0)$ contains both quasiparticle and phonon contributions

Solving for T_v yields $\eta(v)$ in the LO form with

$$v_0 = \sqrt{T_c \kappa(T_0) / \eta_0(0)}$$

Bezugiyj and Shklovskii, Physica C 202, 234 (1992) Kunchur, PRL 89, 137005 (2002) Gurevich and Ciovati, PRB 77, 104501 (2008)

Larkin-Ovchinnikov instability



LO instability of a trapped vortex

Since the LO critical velocity $v_0 \sim 0.1 - 1$ km/s is 1-2 orders of magnitude smaller than velocities of a vortex at H = 10 -100 mT, the LO instability can be essential in SRF cavities.



- What happens to the vortex if its fast tip is LO-unstable while the rest of the vortex is LO-stable?
- Can a vortex be shredded into disconnected pieces by strong surface current?
- Dependence of RF losses and the residual surface resistance caused by trapped vortices on the RF field.
- The extreme vortex dynamics in SRF cavities is not masked by strong overheating typical of dc transport measurements at T << T_c.

Nonlinear dynamic equations for a vortex



Balance of local forces perpendicular to a curvilinear vortex

$$M\dot{v} + \eta(v)v = \epsilon/R - (H_a/\lambda)e^{-x/\lambda}\sin\omega t$$

Dynamic eq. for a dimensionless vertical displacement $u(x,t) = y(x,t)/\lambda, \quad x \to x/\lambda:$

$$\mu \frac{\partial}{\partial t} \left(\frac{\dot{u}}{\sqrt{1 + u'^2}} \right) + \frac{\gamma \dot{u} \sqrt{1 + u'^2}}{1 + u'^2 + \alpha \gamma^2 \dot{u}^2} = \frac{u''}{(1 + u'^2)^{3/2}} - \beta e^{-x} \sin(2\pi t)$$

Takes into account vortex inertia, and nonlinearities of the LO vortex drag and bending rigidity

$$\gamma = f/f_0, \qquad f_0 = H_{c1}\rho_n/H_{c2}\lambda^2\mu_0$$

$$\alpha = (\lambda f_0 / v_0)^2, \qquad \beta = H_a / H_{c1}$$

 $f_0 = 22 \text{ GHz}$ for Nb.

Nonlinear vortex losses and residual resistance

Dissipated power per vortex:

$$p = \int \langle \eta(v)v^2 \rangle ds$$

Surface resistance R_i for the mean trapped flux density B₀ is obtained from $pB_0/\phi_0=R_iH_a^2/2$:

$$R_i(\beta) = \frac{R_0 \gamma^2}{\beta^2} \int_0^1 dt \int_0^l \frac{(1+u'^2)^{1/2} \dot{u}^2 dx}{1+u'^2 + \alpha \gamma^2 \dot{u}^2}, \qquad R_0 = \frac{2\rho_n B_0}{\lambda B_{c2}}$$

For Nb at 1-2 GHz, we have $\gamma \sim 10^{-1}$, and $\alpha \sim 10^2 - 10^4$. At small f and H_a the LO term in the denominator is negligible and R_i is independent of Ha

As H_a and f increase, \dot{u}^2 cancels out and R_i becomes nearly *independent* of frequency and decreases with the RF field:

$$R_i \propto H_a^{-2}$$



LO mechanism of the low-field Q(H) rise

The surface resistance Ri(H) starts decreasing with the field amplitude as the frequency increases. Calculated for different values of

$$\gamma = f/f_0 @ l = 4\lambda, \ \alpha = 3 \cdot 10^3$$

Fit to the experimental data of for a 1.47 GHz Nb cavity Ciovati, JAP 96, 1591 (2004)

$$l = 3\lambda, \;\; B_0 = 0.73 \, \mu T, \ v_0(2K) = 30 \, m/s, \ v_0(1.37K) = 35 \, m/s,$$

Effect of frequency on the field dependence of $R_i(H_a)$





Transition from quasi-harmonic to relaxation

oscillations at the peak in R_i(H). The Campbell

length increases with H_a:

$$L_{\omega}(H_a) \simeq \sqrt{\epsilon/\omega\eta(v)}$$

 $\circ L_{\omega}(H_a, \omega) < l$ before the peak $\circ L_{\omega}(H_a, \omega) > l$ after the peak





- $\circ\,$ Making the surface dirtier and decreasing the impurity mean free path shifts the anomalous drop of the vortex surface resistance with H_a to lower fields.
- May pertain to the low-field Q(H) drop observed on many Nb cavities

Effect of weak overheating on the surface resistance

Thermal feedback for trapped vortices and BCS:

$$\begin{pmatrix} R_i(H_a) + R_0 e^{(T-T_0)\Delta/T_0^2} \end{pmatrix} \frac{H_a^2}{2} = (T-T_0)g,$$
 vortices BCS

Thermal resistance of the cavity wall: $\,g^{-1}=lpha_K^{-1}+d/\kappa\,$





- Interplay of the descending R_i(H) and ascending R₀(H_a) due to overheating produces a minimum in R_s(H_a)
- Too many trapped vortices cause strong overheating which can eliminate the minimum in R_s(H) as v₀(T) increases with T

Could strong pinning be effective in SRF cavities?



GOOD

- Artificial pinning centers (APCs) which take 10% of current-carrying cross-section can produce critical current densities $J_s \simeq 0.1 J_d$
- For cavities this can only be effective below the depinning field $H < 0.1H_c = 20 \text{ mT} = 10\%$ of the SRF breakdown field for Nb.
- Reduction of vortex losses only in a small low-H part of the field operation range

 α -Ti ribbons in a Nb-Ti alloy (D. Larbalestier & P. Lee)

BAD

- 10% of metallic APCs produce huge ohmic losses above the proximity effect breakdown field. Incompatible with high Q controlled by the BCS surface resistance
- 10% of dielectric APCs block the current-carrying cross section, greatly increasing the field penetration depth and the BCS surface resistance
- Above the depinning field, high Bean's hysteretic losses make high-J_c SRF cavities no better than the normal Cu cavities

Conclusions

- At RF fields of 100-200 mT tips of vortices trapped in Nb cavities can reach velocities of a few km/s approaching the speed of sound (3.5 km/s)
- Extreme nonlinear dynamics of the elastic vortex, drastic change of a hot moving vortex core, strong pairbeaking effects and nonequilibrium kinetics of quasiparticles.
- Decrease of the residual surface resistance due to the Larkin-Ovchinnikov mechanism and electron overheating in the vortex core.
- The descending field dependence of the surface Ri(H) develops as the frequency increases.
 A new mechanism of the Q(H) rise which can be tuned by impurities.
- Pinning at the surface can only reduce vortex dissipation at low RF fields << H_s.
- High-Q SRF cavities offer a unique opportunity to investigate the extreme dynamics of vortices at low temperatures.